

MAE/MSE 502 Spring 2020 HW5 Solution

Prob 1

$$K = \frac{10}{3}$$

Prob 2

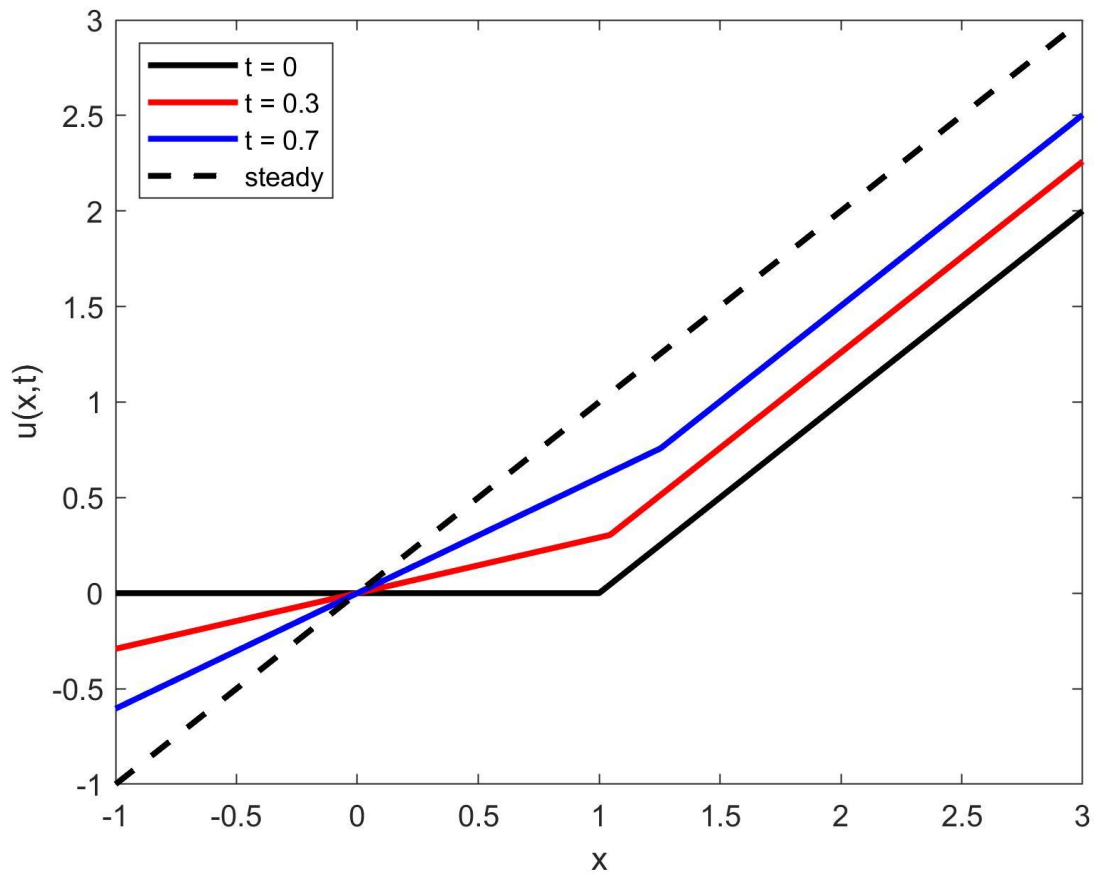
$$u(x, t) = \frac{1}{1+t} e^{-\left(\frac{x}{1+t}\right)^2 + t}$$

Prob 3

$$u(x, t) = \begin{cases} x \tanh(t), & \text{if } x < \cosh(t) \\ x - e^{-t}, & \text{if } x \geq \cosh(t) \end{cases}$$

Steady solution: $u_s(x) = x$.

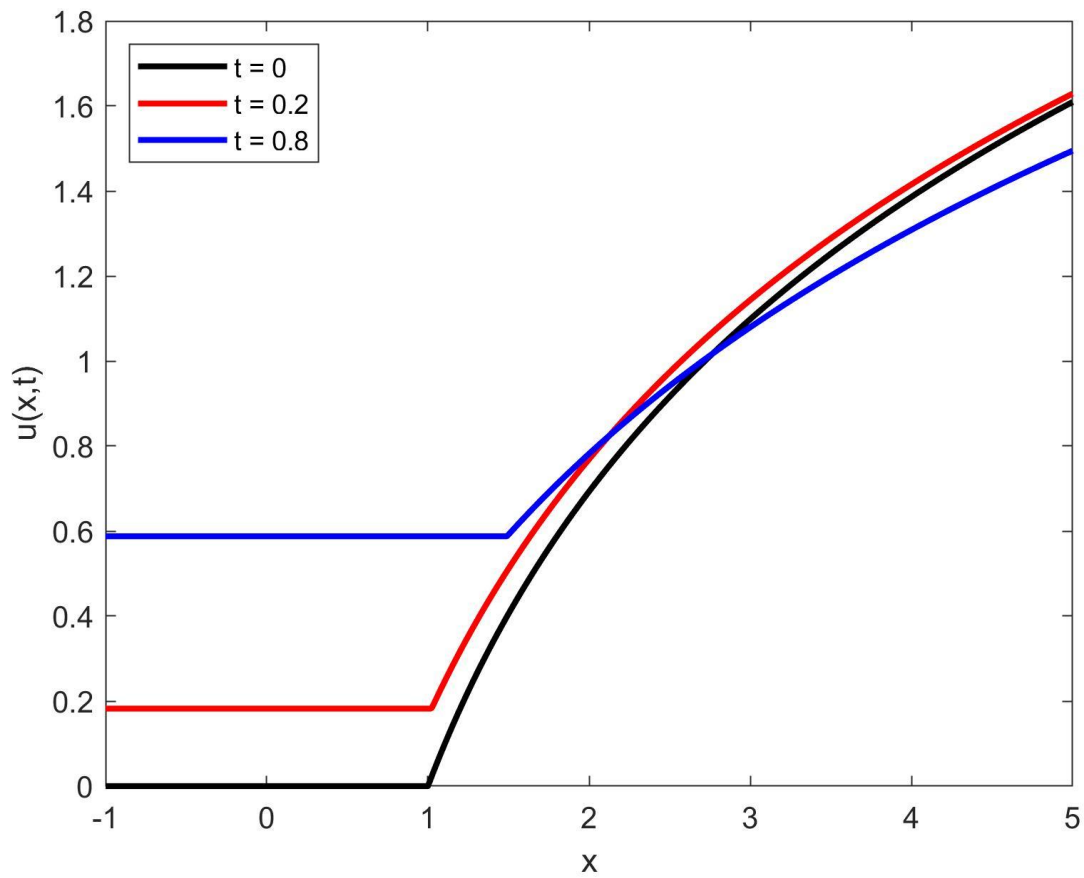
Plot:



Prob 4

$$u(x, t) = \begin{cases} \ln(t + 1), & \text{if } x < 1 + \frac{t^2}{2} + \frac{t^3}{3} \\ \ln \left[t + \frac{\left(x - \frac{t^3}{3}\right)}{\left(1 + \frac{t^2}{2}\right)} \right], & \text{if } x \geq 1 + \frac{t^2}{2} + \frac{t^3}{3} \end{cases}$$

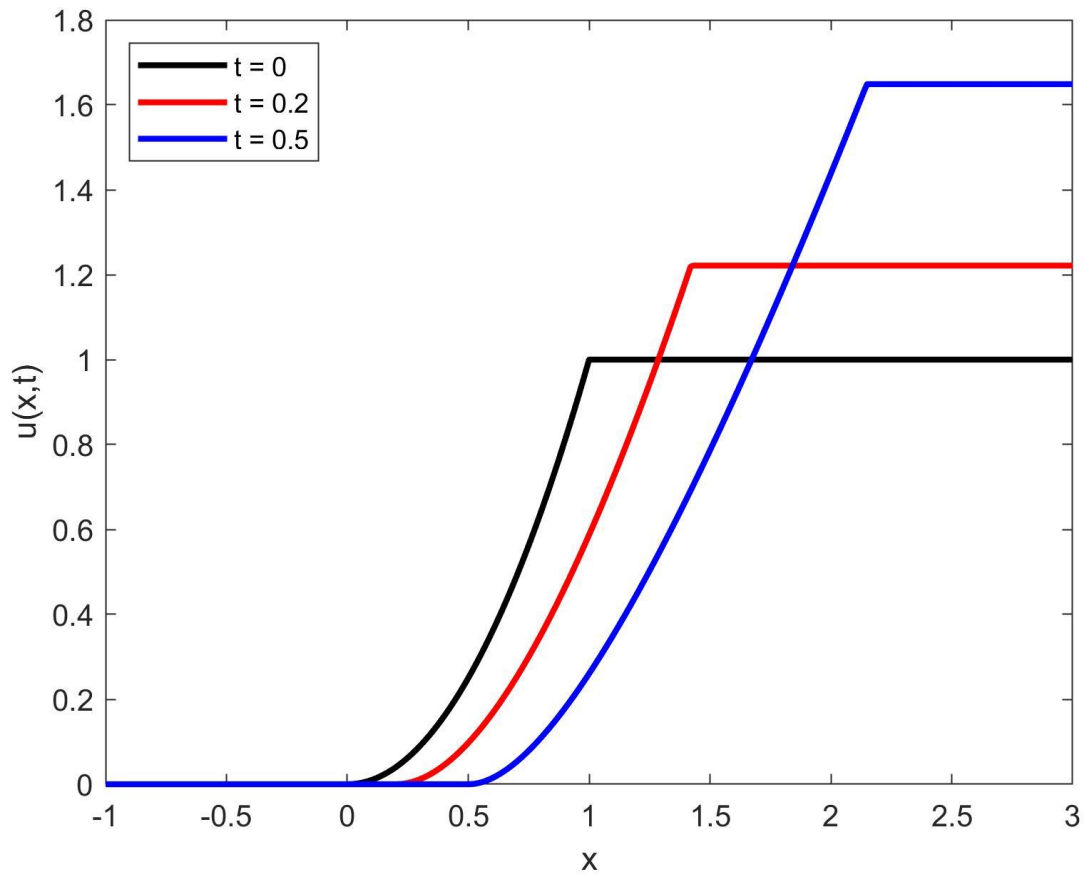
Plot:



Prob 5

$$u(x,t) = \begin{cases} 0, & \text{if } x < t \\ \left[\frac{-1 + \sqrt{1 - 4(e^t - 1)(t - x)}}{2(e^t - 1)} \right]^2 e^t, & \text{if } t \leq x \leq t + e^t \\ e^t, & \text{if } x > t + e^t \end{cases}$$

Plot:



Prob 6

$$u(x,t) = x(e^t - 1) + (2-t)e^t - t - 1$$