MAE/MSE 502 Fall 2021 HW1 Solution

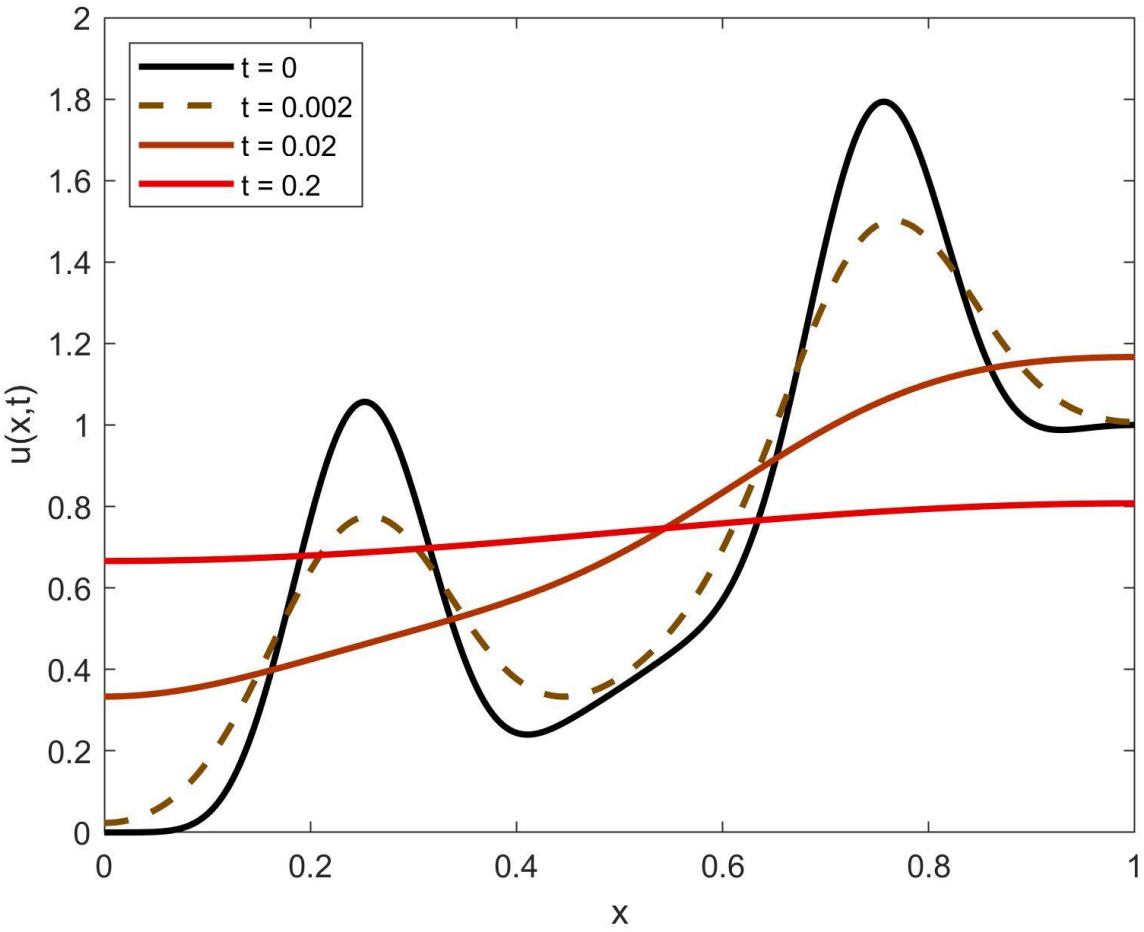
Problem 1(a)

$$u(x,t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x) \exp(-(n\pi)^2 t)$$

where

$a_0 = \int_0^1 P(x)\, dx$, and $a_n = 2\int_0^1 P(x) \cos(n\pi x) dx$ for $n > 0$.

Plot:



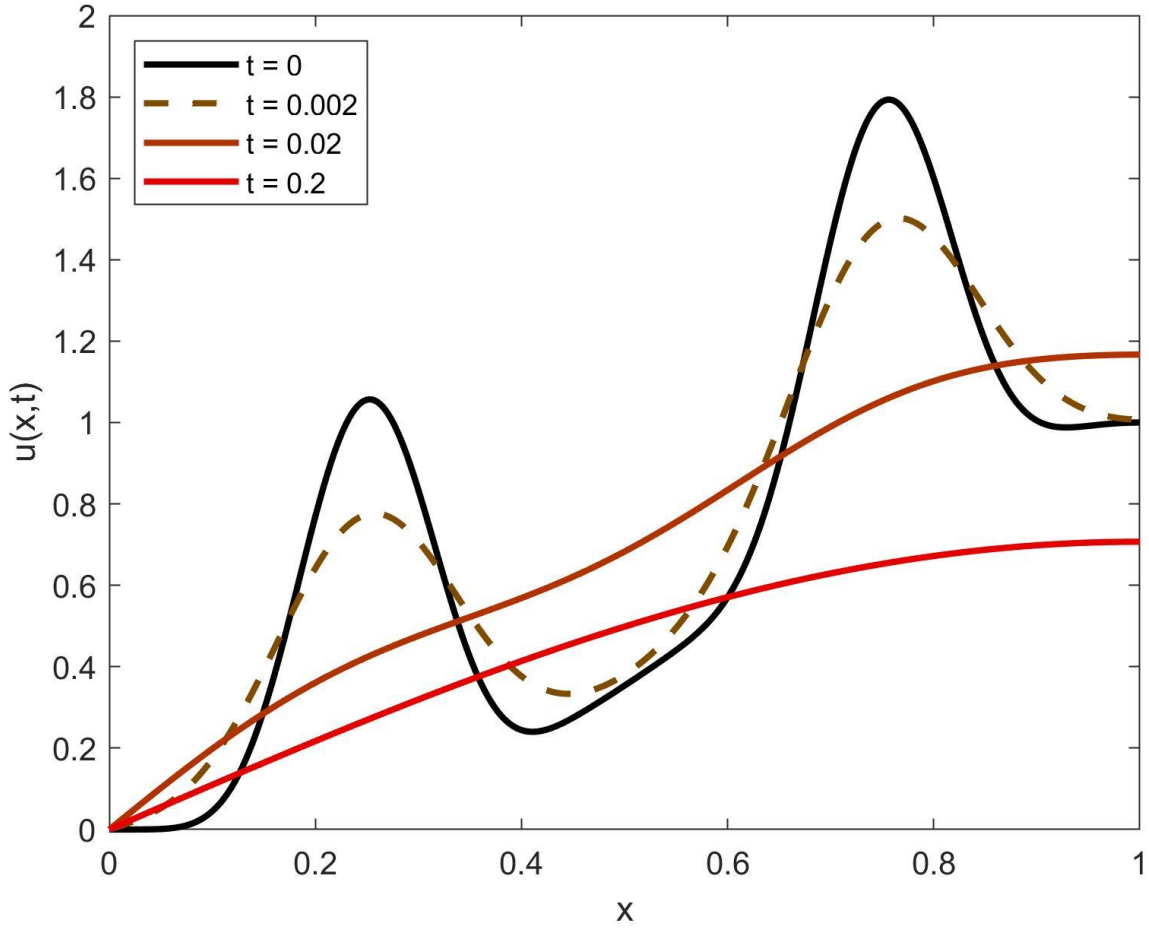
Problem 1(b)

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{2}\right) \exp\left(-\left(\frac{n\pi}{2}\right)^2 t\right)$$

where the summation is over odd values of n only, and

$a_n = 2\int_0^1 P(x)\,\sin(\frac{n\pi x}{2})dx$, for odd values of n.

Plot:

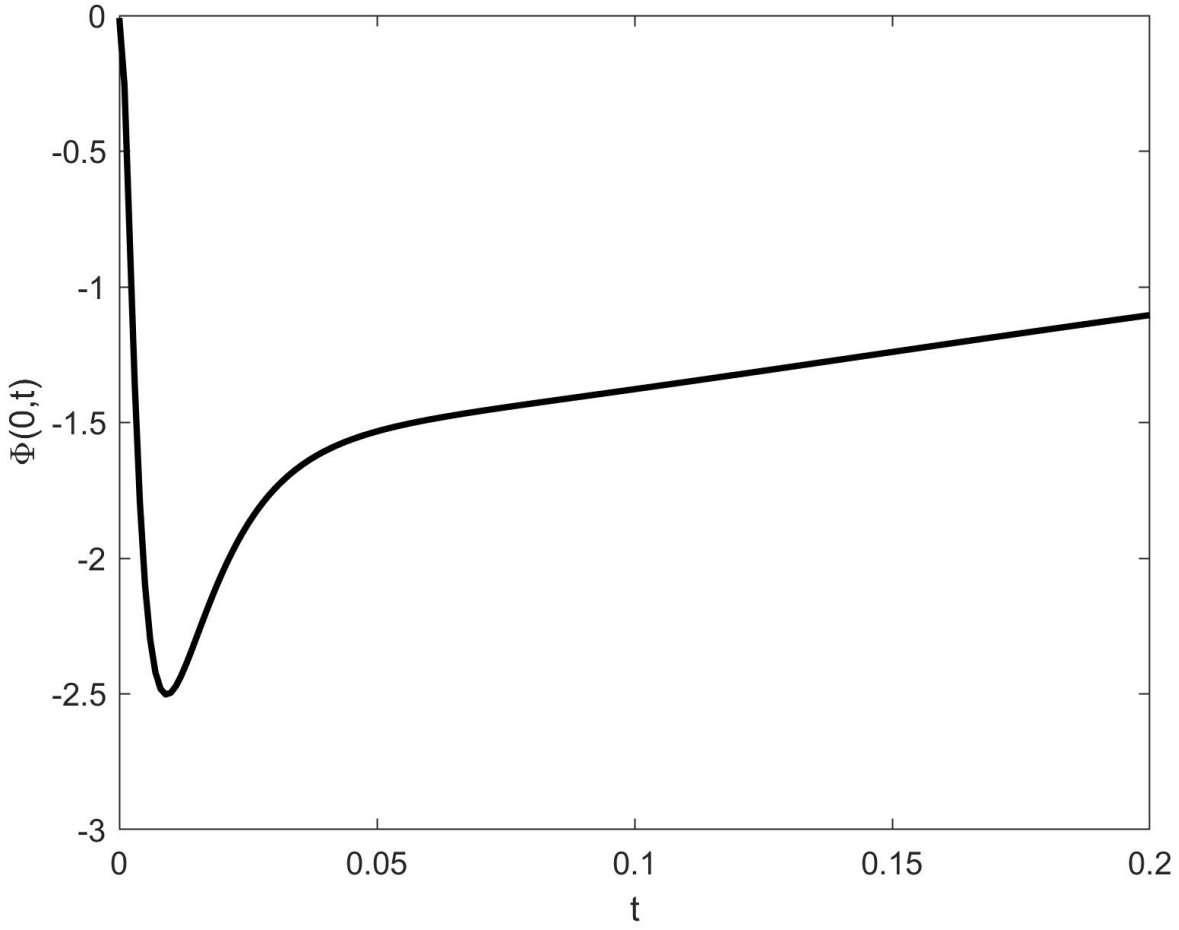


Problem 1(c)

$$\Phi(0,t) = -\sum_{n=1}^{\infty} a_n \left(\frac{n\pi}{2}\right) \exp\left(-\left(\frac{n\pi}{2}\right)^2 t\right)$$

where the summation is over odd values of *n* only, and the expansion coefficients { a_n } are the same as in Part (b).

Plot:



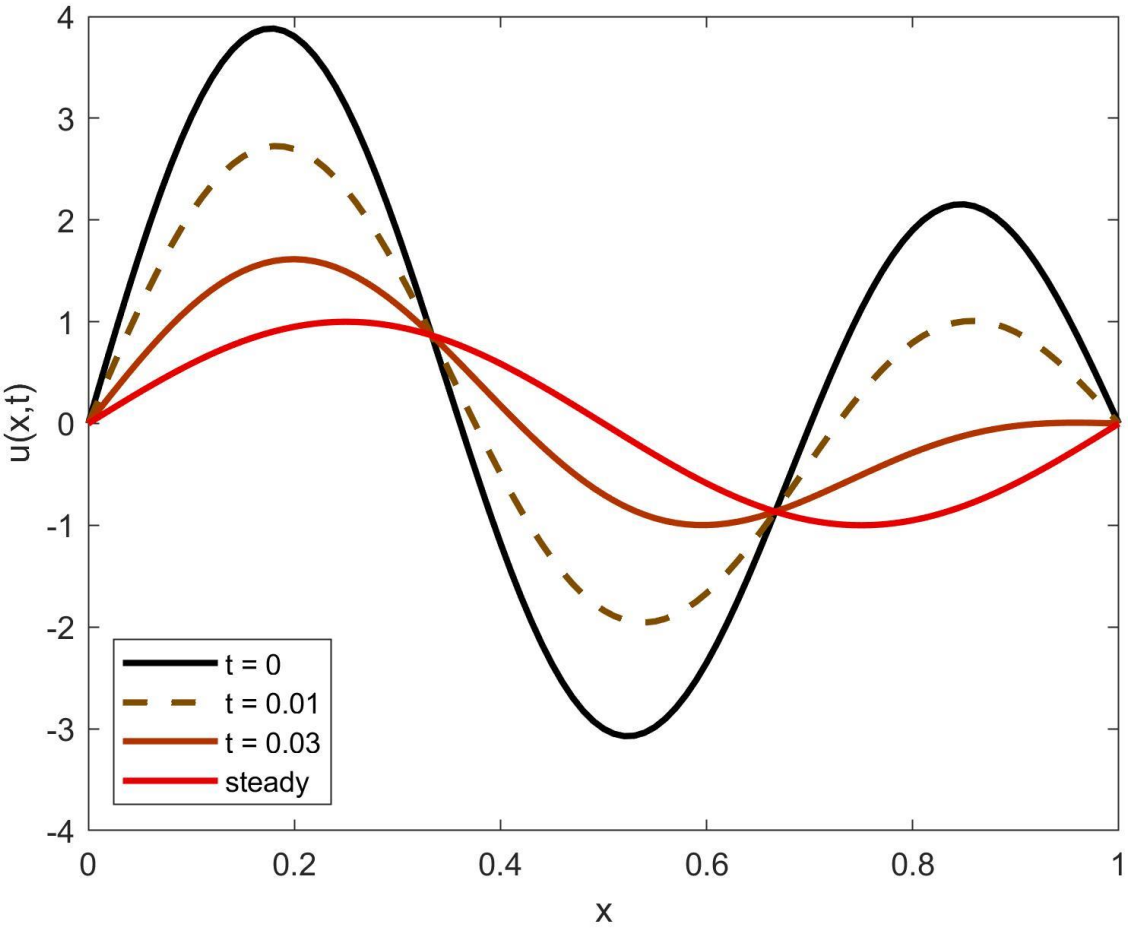
Problem 2

$$u(x,t) = \sin(2\pi x) + 3\sin(3\pi x)\, e^{-5\pi^2(1-e^{-t})}$$

Steady solution is

$$u_s(x) = \sin(2\pi x) + 3\sin(3\pi x)\, e^{-5\pi^2}$$

Plot:



Problem 3

$$u(x,t) = e^{-t^2/2} + \cos(\pi x)\,(1+t)^{-\pi^2} e^{-t^2/2}$$

Problem 4

We will discuss the solution of this problem in class.