MAE/MSE 502, Fall 2021 HW2 Solution

Problem 1

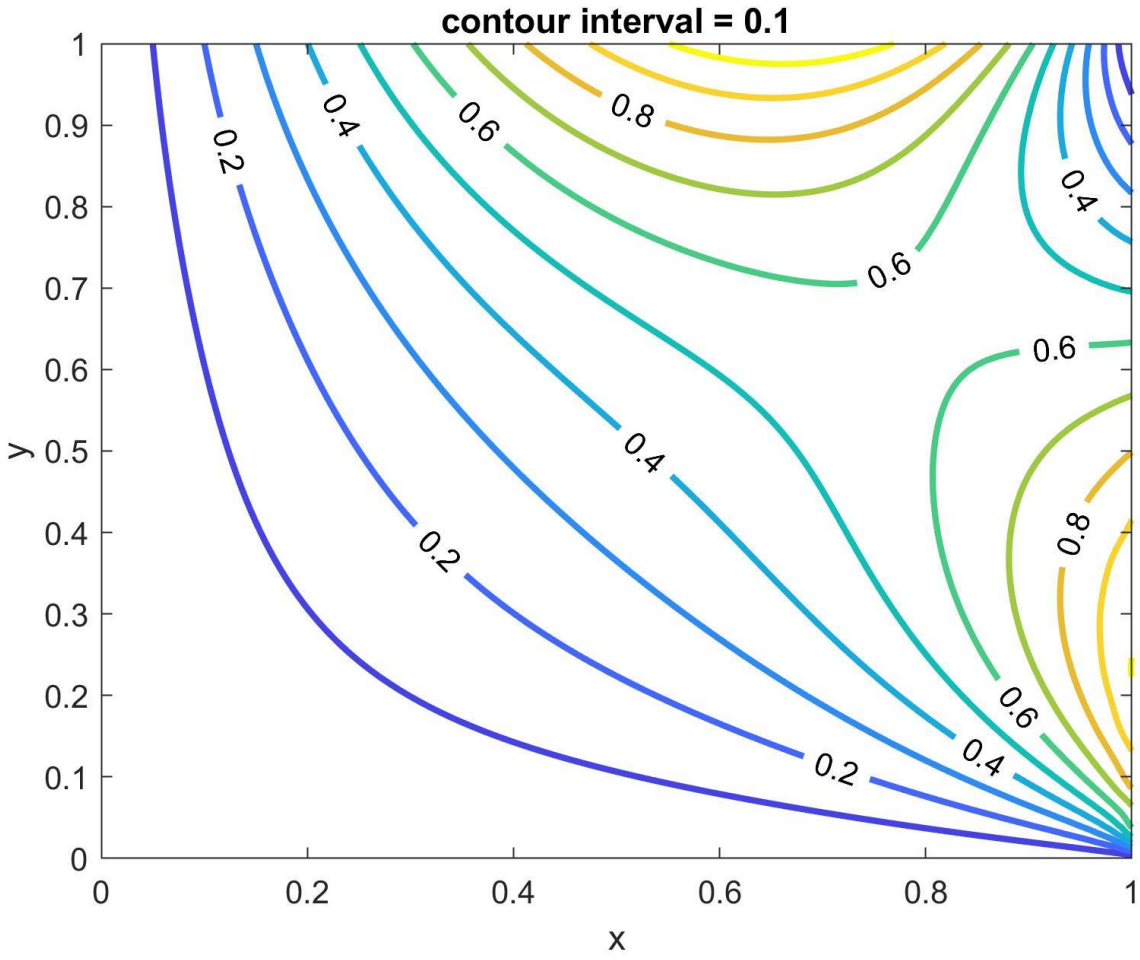
$$u(x,y) = \sum_{n=1}^{\infty} a_n \sin(n\pi y)\sinh(n\pi x) + b_n \sin(n\pi x)\sinh(n\pi y)$$

where

$$a_n = \frac{2}{\sinh(n\pi)} \int_0^1 \sin\left(\pi\sqrt{y}\right)\sin(n\pi y)\, dy$$

$$b_n = \frac{2}{\sinh(n\pi)} \int_0^1 2(x - x^5)\sin(n\pi x) dx$$

Plot:



Problem 2

(a) Solvability condition is satisfied. There are infinitely many solutions

(b)

$$u(x,y) = C + 2y + \frac{\cosh(\pi y)\cos(3\pi x)}{\pi \sinh(\pi)}$$

where *C* is an arbitrary constant (which corroborates the conclusion from Part (a)).

Problem 3

$$u(x,y) = e^{4x} + e^{5x}\cos(2y)$$

Problem 4

$$u(x,y) = \frac{\sin(y)}{\sin(1)} + (y+1)\cos(x) + \frac{\sinh(\sqrt{3}\,y)}{\sinh(\sqrt{3})}\cos(2x)$$

The detailed procedure for Problem 2(b), 3, and 4 will be posted separately.