MAE/MSE 502, Fall 2021 Homework #4

Please follow the rules on collaboration as given in the document for Homework #1. Your work should include a statement of collaboration.

For all problems in this homework, we expect a closed-form exact solution with only a finite number of terms and without any unevaluated integral. The solution, $u(x,t)$, should be expressed explicitly in real functions of x and t and real numbers. Expect a deduction if these requirements are not satisfied.

Problem 1 (2.5 points)
For $u(x,t)$ defined on the domain of $0 \le x \le \pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + t^2\, e^{-t} \cos(x) + \cos(t) + 1$$

with the boundary conditions

(i) $u_x(0,t) = 0$ (ii) $u_x(\pi,t) = 0$ (iii) $u(x,0) = \cos(x)$.

Problem 2 (2.5 points)
For $u(x,t)$ defined on the domain of $0 \le x \le \pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 2\sin(x)$$

with the boundary conditions,

(i) $u(0,t) = 1$ (ii) $u_x(\pi,t) = -2$ (iii) $u(x,0) = 1 + 2\sin(x) + \sin(x/2)$

Problem 3 (2.5 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^4 u}{\partial x^4} + \cos(x)\cos(t)$$

with periodic boundary conditions in x-direction, and the boundary condition in t-direction given as

(i) $u(x,0) = \sin(x) + \sin(3x)$

Problem 4 (2.5 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, solve the PDE

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = \pi^2 \sin(\pi x)$$

with the boundary conditions

(i) $u(0,t) = 0$ (ii) $u(1,t) = 0$ (iii) $u(x,0) = \sin(\pi x)$ (iv) $u_t(x,0) = \sin(\pi x)$