MAE/MSE 502, Fall 2021 Homework #5

For all problems in this homework, we expect a closed-form solution without any unevaluated integrals. The solution, $u(x, t)$, must be written explicitly as a function of x and t.

Problem 1 (1.5 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$(1+t)\frac{\partial u}{\partial t} + t\,x\,\frac{\partial u}{\partial x} = t\,u$$

with the boundary condition,

$$u(x,0) = e^{-x^2}.$$

Problem 2 (3 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\frac{\partial u}{\partial t} + (1+u)\,\frac{\partial u}{\partial x} = u$$

with the boundary condition,

$$u(x,0) = \begin{cases} 0\,, & if\ x < 0 \\ 2x, & if\ 0 \leq x \leq 1 \\ 2\,, & if\ x > 1 \end{cases}$$

Plot the solution, $u(x, t)$, as a function of x at $t = 0$, 0.3 and 0.6. The recommended range for plotting is $-1 \leq x \leq 4$. Please collect all three curves in a single plot.

Problem 3 (3 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\frac{\partial u}{\partial t} + 2\,e^{u}\,\frac{\partial u}{\partial x} = e^{-u}$$

with the boundary condition,

$$u(x,0) = \begin{cases} 0\,, & if\ x < 0 \\ ln(x+1)\ , & if\ x \geq 0 \end{cases}$$

($ln(x)$ is the natural logarithm of x.)
Plot the solution, $u(x, t)$, as a function of x at $t = 0$, 0.3 and 0.6. The recommended range for plotting is $-1 \leq x \leq 4$. Please collect all three curves in a single plot.

Problem 4 (2.5 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$$

with the boundary conditions,

(i) $u(x,0) = x$

(ii) $u_t(x,0) = x$.

In addition to providing the full solution, answer the following question: You will find that the solution can be expressed in the form of $u(x, t) = A(t)\, x + B(t)$. (In other words, at any given t, the solution is a straight line.) What are the values of $A(t)$ and $B(t)$ at $t = 0.5$? [We ask this question to speed up the process of grading. Instructor will first check these two numbers. Detailed procedure will be checked only if these two numbers are incorrect (or missing).]

[Note: If it might be useful, $\int_0^t t\, e^{-t}\, dt = 1 - e^{-t}(1+t)$.]