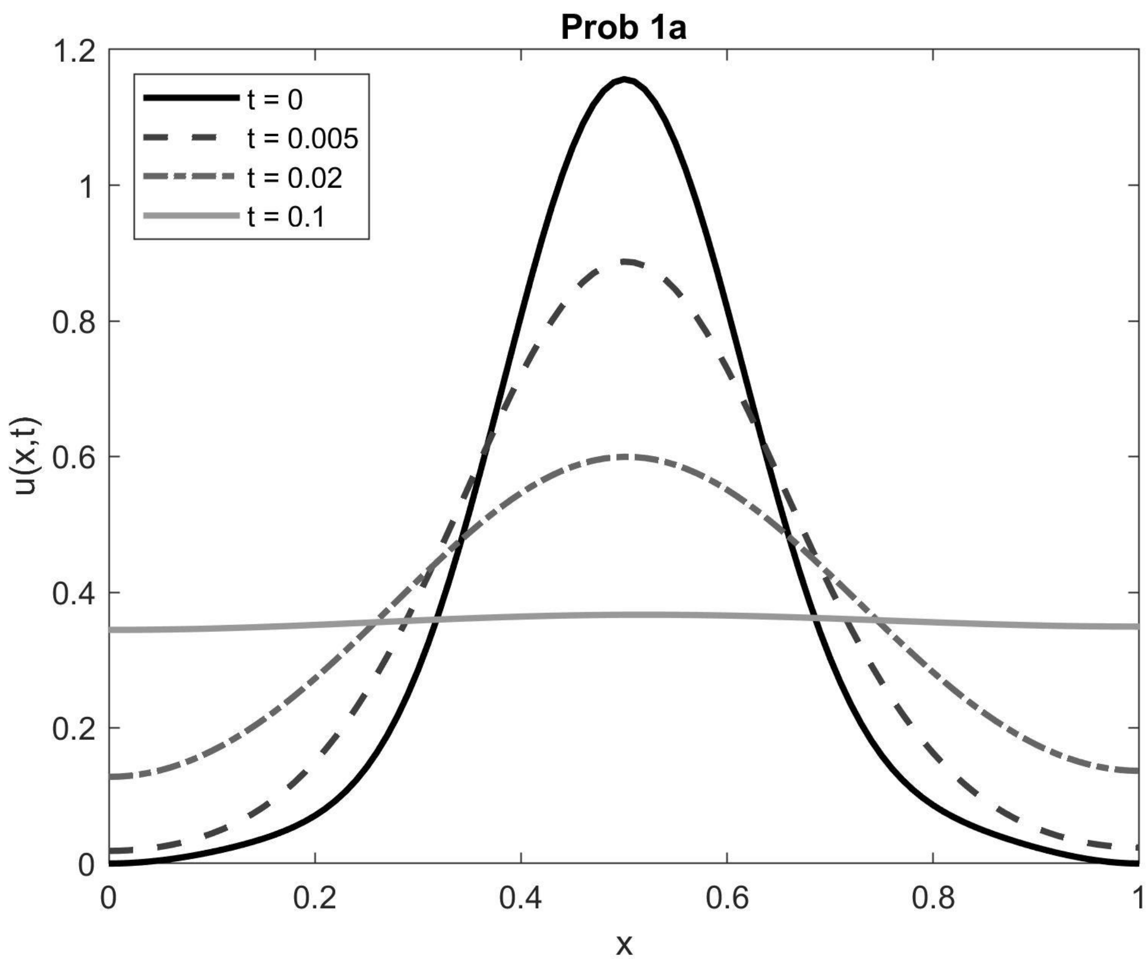
MAE/MSE 502 Spring 2021 HW1 Solution

Prob 1(a)

$$u(x,t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x) \exp(-(n\pi)^2 t)$$

where

$a_0 = \int_0^1 P(x)\, dx$, and $a_n = 2\int_0^1 P(x)\cos(n\pi x) dx$ for $n > 0$,

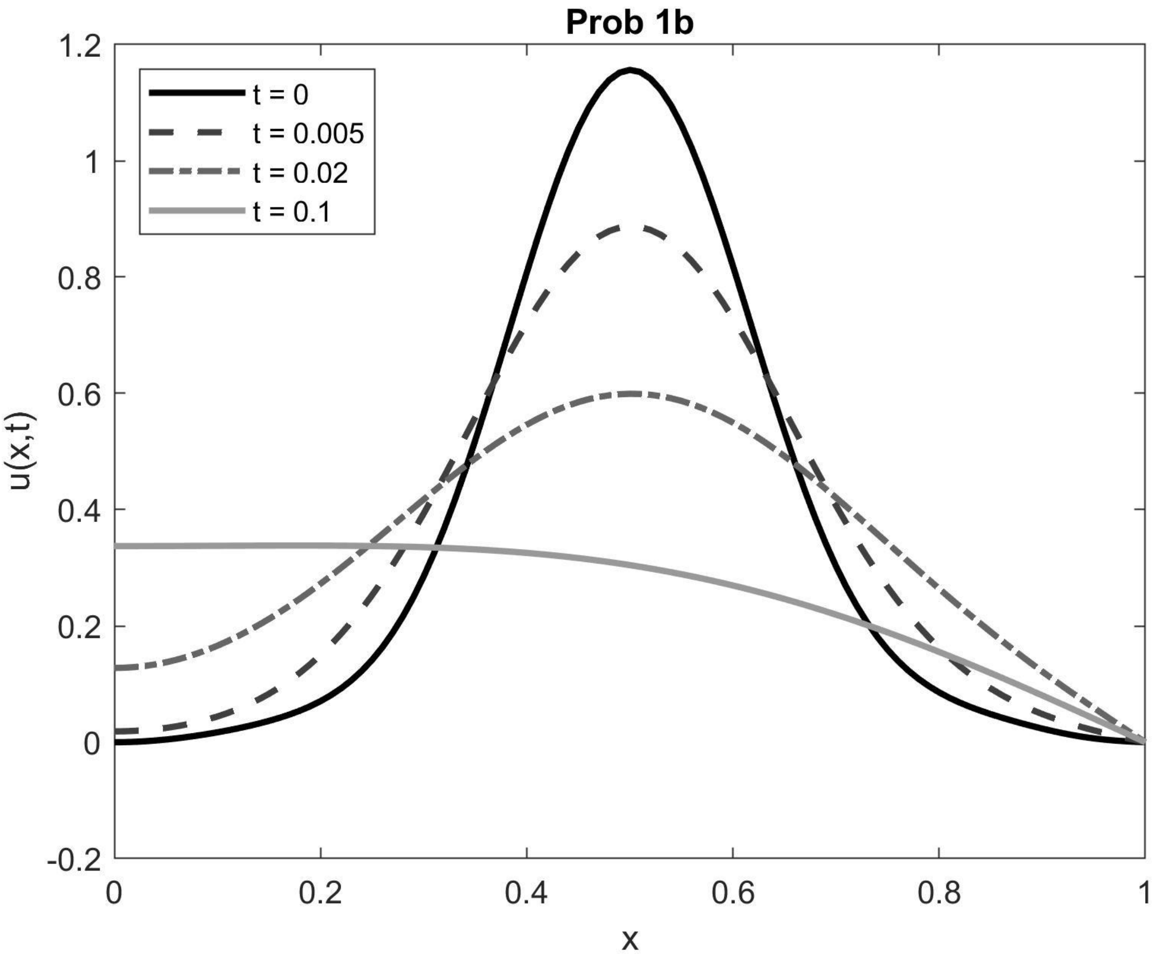


Prob 1(b)

$$u(x,t) = \sum_{n=1}^{\infty} a_n \cos(\frac{n\pi x}{2}) \exp(-(\frac{n\pi}{2})^2 t)$$

where the summation is over odd values of n only, and

$$a_n = 2\int_0^1 P(x)\cos(\frac{n\pi x}{2})dx$$, for odd values of n.

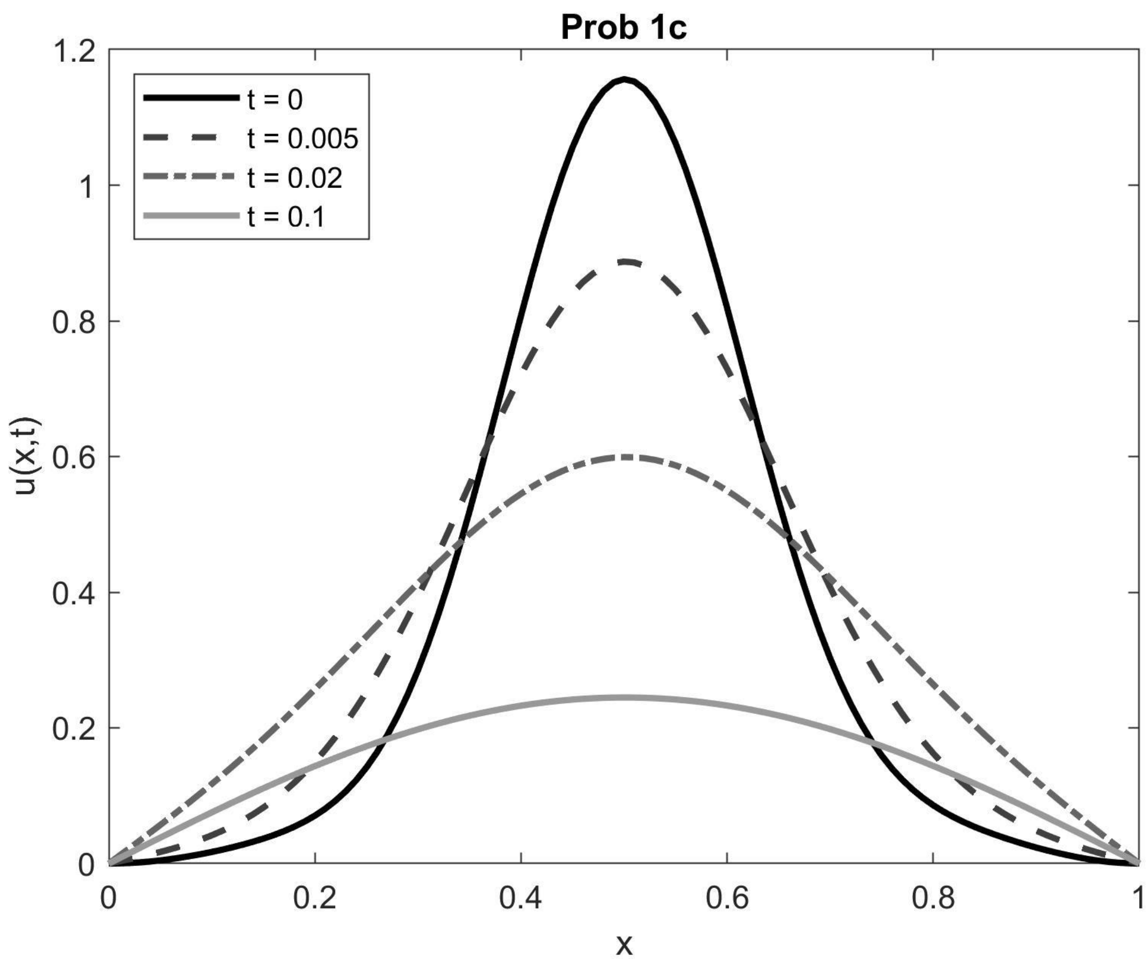


Prob 1(c)

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin(n\pi x)\exp(-(n\pi)^2 t)$$

where

$a_0 = \int_0^1 P(x)\,dx$, and $a_n = 2\int_0^1 P(x)\sin(n\pi x)dx$ for $n > 0$,



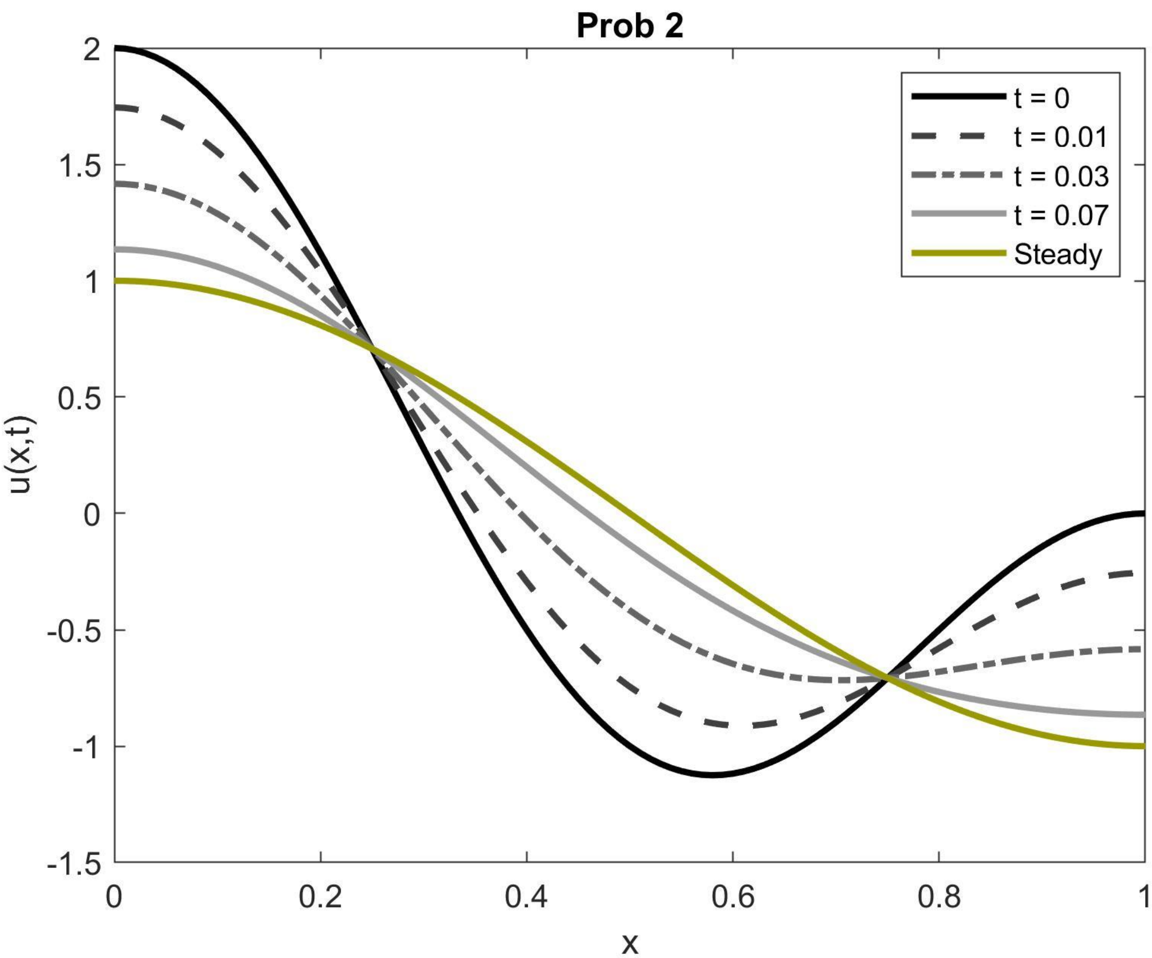
Prob 2

$$u(x,t) = \cos(\pi x) + \cos(2\pi x)\,(1+t)^{-3\pi^2}$$

Steady solution is

$$u_s(x) = \cos(\pi x)$$

Plot:



Prob 3

$$u(x,t) = \sin(5x)\,e^{3t/4} + \sin(10x) + \sin(20x)\,e^{-3t}$$

The solutions for Prob 4 and Prob 5 have been given in class (Lecture 13).