MAE/MSE 502, Spring 2021 HW2 Solution

Problem 1

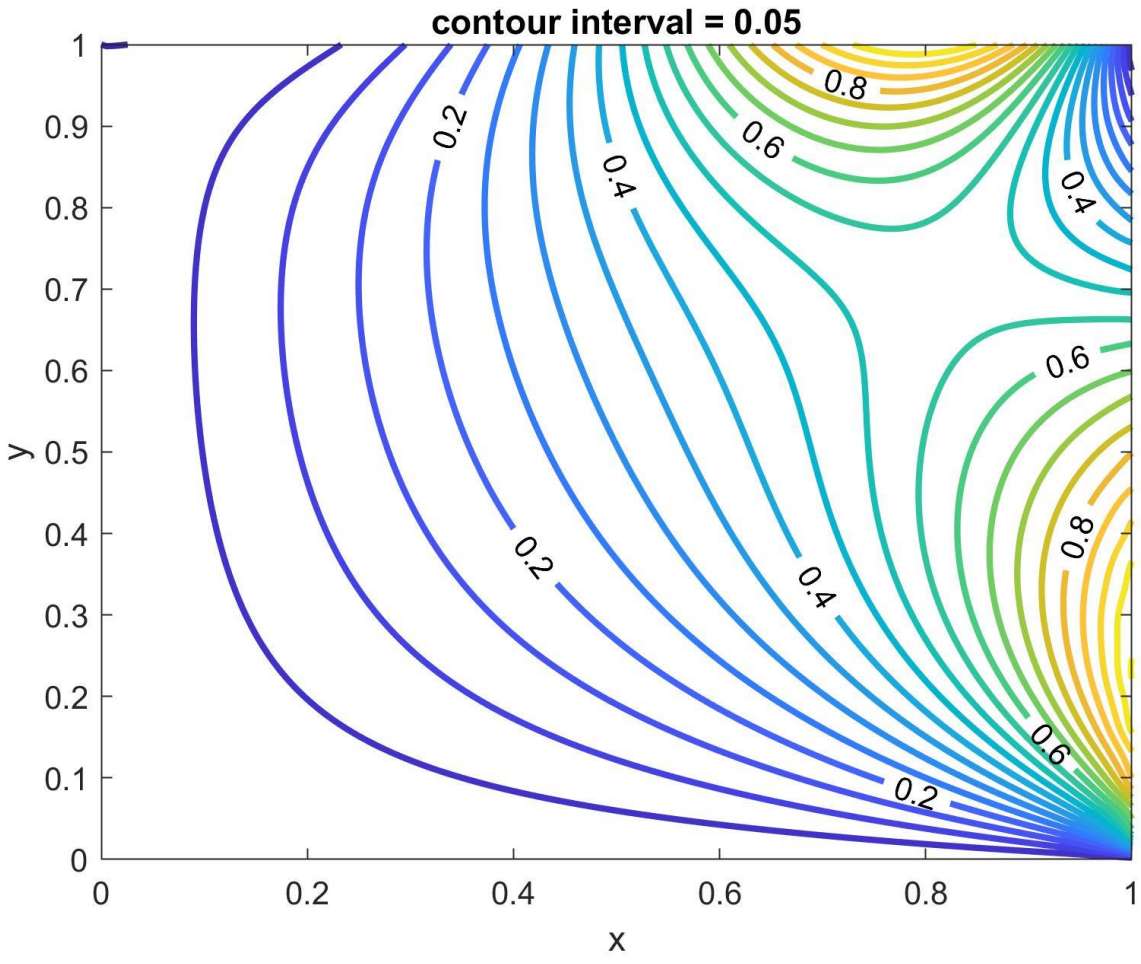
$$u(x,y) = \sum_{n=1}^{\infty} a_n \sin(n\pi y)\sinh(n\pi x) + b_n \sin(n\pi x)\sinh(n\pi y)$$

where

$$a_n = \frac{2}{\sinh(n\pi)} \int_0^1 \sin\left(\pi\sqrt{y}\right)\sin(n\pi y)\, dy$$

$$b_n = \frac{2}{\sinh(n\pi)} \int_0^1 4\,(x^3 - x^6)\sin(n\pi x) dx$$

Plot:



Problem 2

(a) There are infinitely many solutions

(b)

$$u(x,y) = C + 100y + \frac{\cosh(100\pi y)\cos(100\pi x)}{100\pi \sinh(100\pi)}$$

where *C* is an arbitrary constant (which corroborates the conclusion from Part (a)).

Problem 3

$$u(x,y) = \cos(4y) - \cot(4)\sin(4y) + (1-y)\cos(4x) + \frac{\sinh(3y)\cos(5x)}{\sinh(3)}$$

Problem 4

$$u(x,y) = (16\,e^{-x} - e^{3x})\sin(2y)$$

Problem 5

A = –16