

Problem 1

$$u(x, t) = \sum_{n=1}^{\infty} a_n \sin(n\pi x) \cos(n\pi t)$$

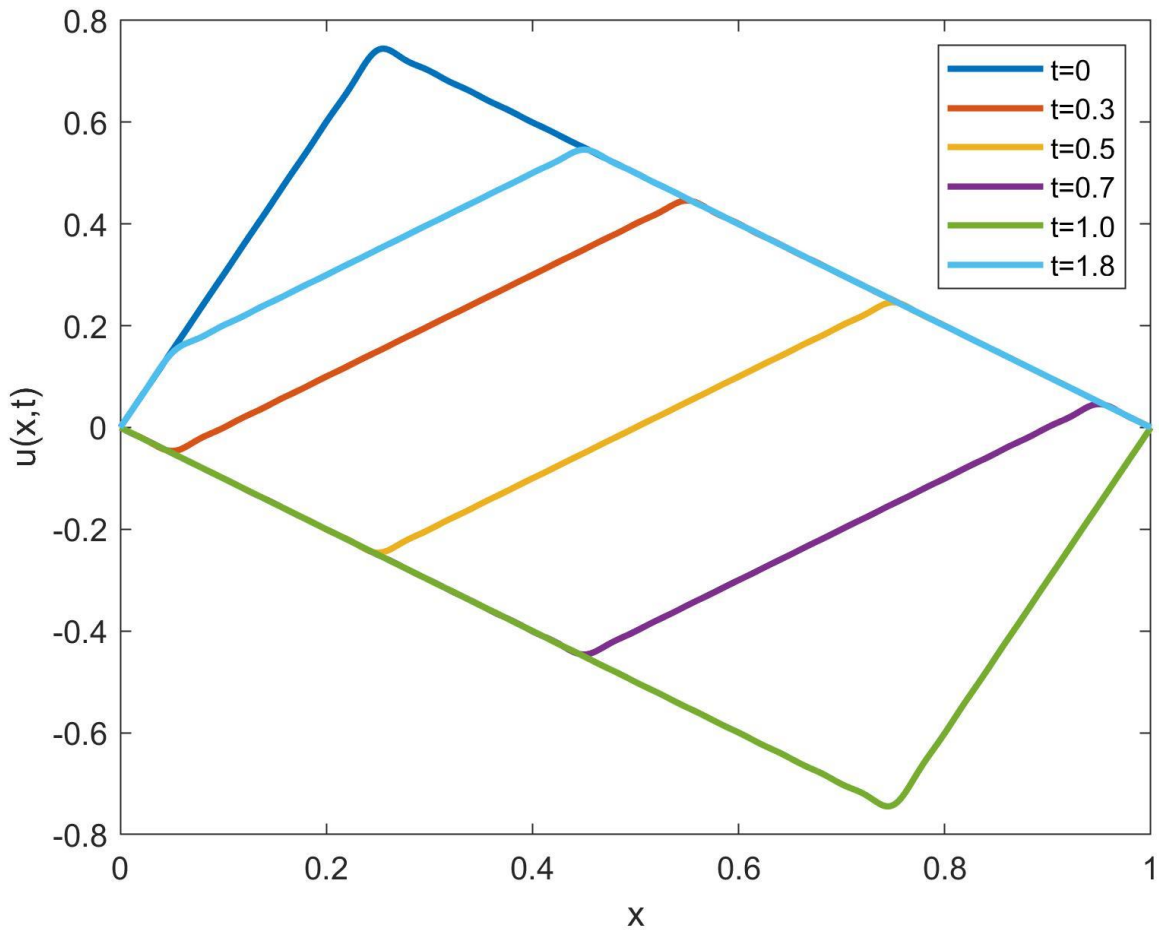
For Part (a):

$$a_n = 2 \left[\int_0^{0.25} 3x \sin(n\pi x) dx + \int_{0.25}^1 (1-x) \sin(n\pi x) dx \right].$$

For Part (b):

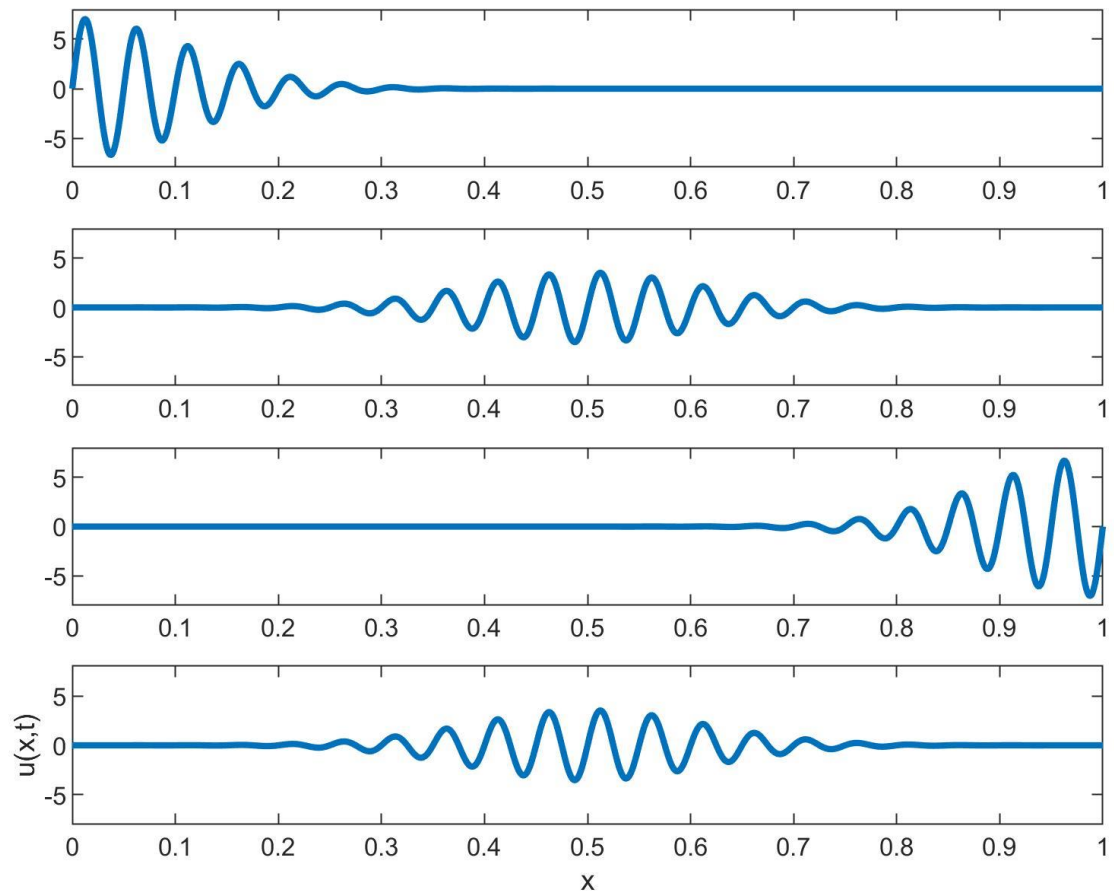
$$a_n = \exp\left[-\left(\frac{n-40}{4}\right)^2\right], \text{ for } 30 \leq n \leq 50, \text{ and } a_n = 0 \text{ otherwise}$$

Plot for Q1a:



Plot for Q1b:

Top to bottom: $t = 0, 0.5, 1, 1.5$



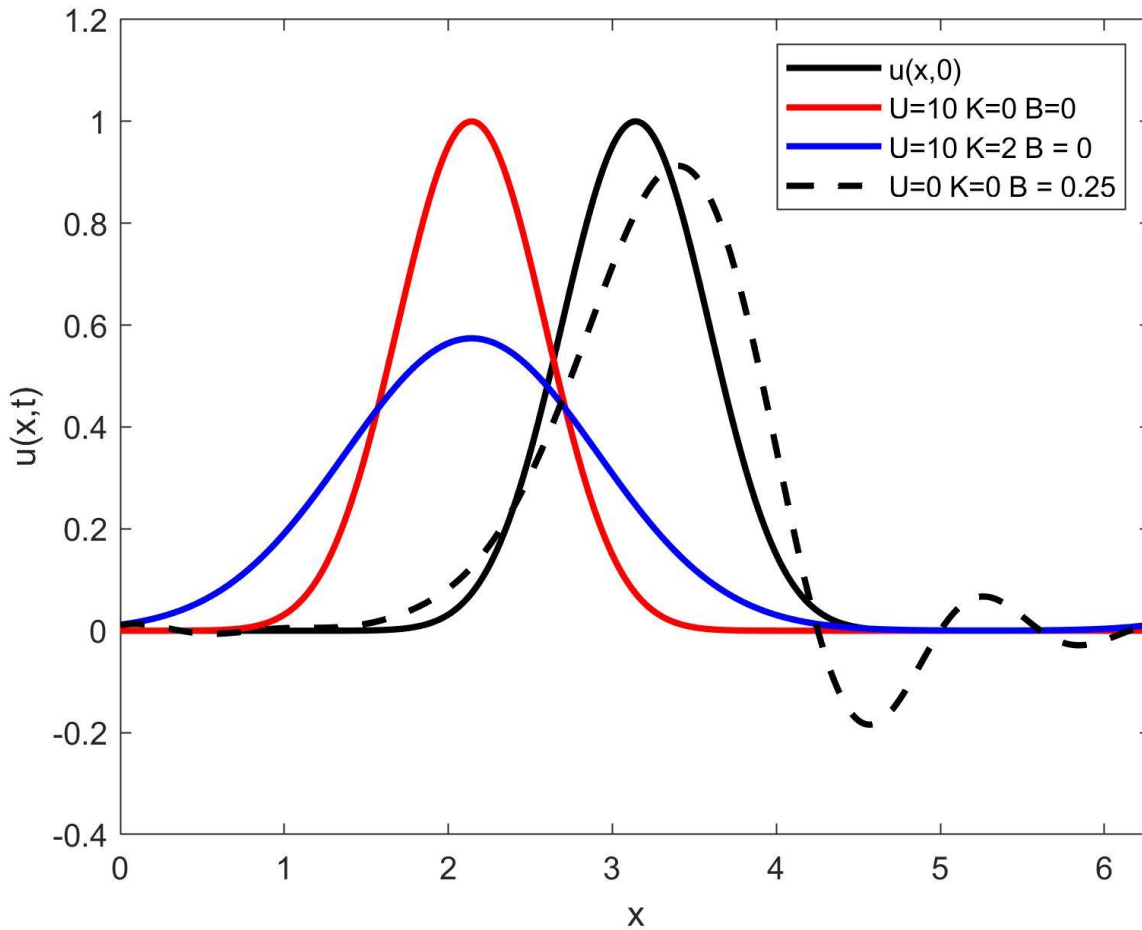
Problem 2

$$u(x, t) = \sum_{n=-\infty}^{\infty} C_n(0) e^{(inU - n^2K - in^3B)t + inx}$$

where

$$C_n(0) = \frac{1}{2\pi} \int_0^{2\pi} u(x, 0) e^{-inx} dx$$

Plot:



Problem 3

$$u(x, t) = (1 + t)^4 + \cos(2x + 2 \ln(1 + t)) + \sin(2x + 2 \ln(1 + t))$$

Problem 4

$$u(x, t) = e^{-t} + \sin\left(x - \frac{t^2}{2}\right)$$

Problem 5

$$u(x, t) = \cosh(2t) + \frac{1}{2} \sin(2x + 2t) - \frac{1}{2} \sin(2x)$$