See Homework #1 for rules on collaboration.

Problem 1 (2 points)
For $u(x, t)$ defined on the domain of $0 \leq x \leq 1$ and $t \geq 0$, solve the PDE,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - \pi^2 u + e^{-\pi^2 t} + \cos(\pi x) e^{-2\pi^2 t}$$

with the boundary conditions,

(i) $u_x(0, t) = 0$,   (ii) $u_x(1, t) = 0$, (iii) $u(x, 0) = 1 + \cos(\pi x)$

We expect a closed form solution that contains only a finite number of terms and with no unevaluated integrals.

Problem 2 (3 points)
For $u(x,t)$ defined on the domain of $0 \leq x \leq 2\pi$ and $t \geq 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \cos(x)$$

with the boundary conditions,

(i) $u(0, t) = 0$ (ii) $u(2\pi, t) = 2\pi$ (iii) $u(x, 0) = \sin(0.5\,x) + \cos(x) + x - 1$

We expect a closed form solution that contains only a finite number of terms and with no unevaluated integrals.

Problem 3 (3 points)
For $u(x, t)$ defined on the domain of $0 \leq x \leq 2\pi$ and $t \geq 0$, solve the PDE,

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^4 u}{\partial x^4} + t + \cos(2x) + \sin(x)\cos(t)$$

with periodic boundary conditions in x-direction, and the boundary conditions in the t-direction given as

(i) $u(x, 0) = 0$ (ii) $u_t(x, 0) = 0$

We expect a closed form solution that contains only a finite number of terms and with no unevaluated integrals.

Problem 4 (3 points)
For $u(x,t)$ defined on the domain of $0 \leq x \leq 2\pi$ and $t \geq 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u$$

with the boundary conditions,

(i) $u_x(0, t) = 1$ (ii) $u(2\pi, t) = 2$ (iii) $u(x, 0) = 2\cos(x) + \sin(x) + \cos(0.25\,x)$ (u_x is $\partial u/\partial x$).

We expect a closed form solution that contains only a finite number of terms and with no unevaluated integrals.