MAE/MSE 502, Spring 2021, HW5 Solution

Problem 1

$$u(x,t) = \frac{1}{1+t}\ exp(-[x(1+t)e^{-t}]^2 + t)$$

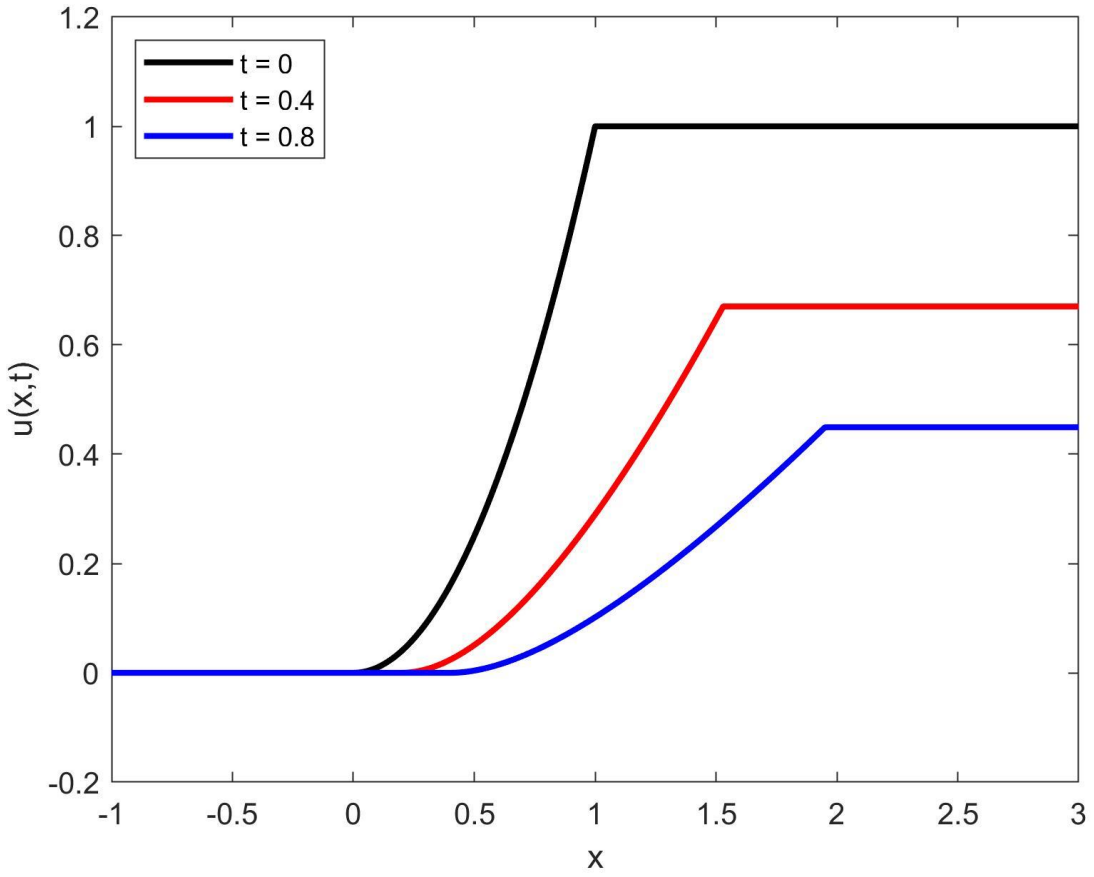
Problem 2

$$u(x,t) = exp\left(-\left[\left(x - \frac{yt}{1+t}\right)^2 + \left(\frac{y}{1+t}\right)^2\right] + t\right)$$

Problem 3

$$u(x,t) = \begin{cases} 0\,, & if\ x < 0.5t \\ e^{-t}\left\{\dfrac{-1+\sqrt{1-4(0.5t-x)(1-e^{-t})}}{2\,(1-e^{-t})}\right\}^2, & if\ 0.5t \le x \le 2+0.5t-e^{-t} \\ e^{-t}, & if\ x > 2+0.5t-e^{-t} \end{cases}$$

Plot:

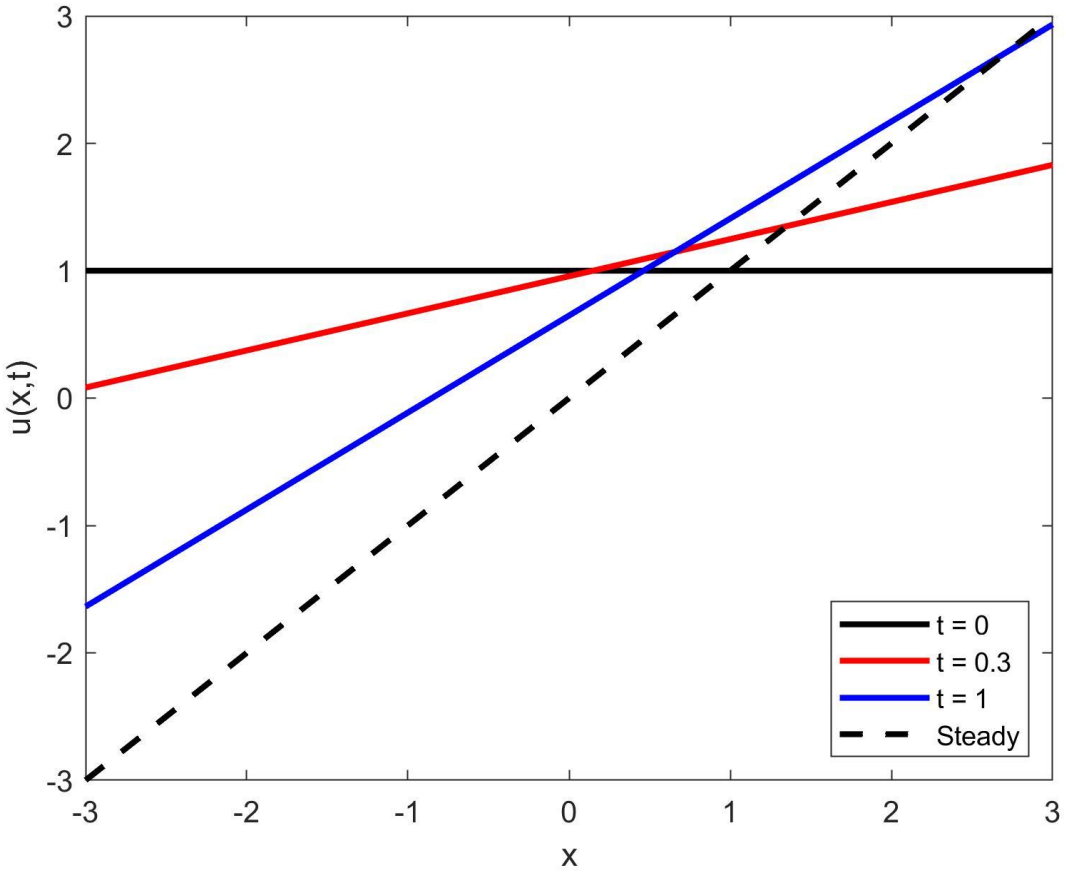


Problem 4

$$u(x,t) = \frac{1 + x\sinh(t)}{\cosh(t)}$$

Steady solution is $u_s(x) = x$.

Plot:



Problem 5

$$u(x,t) = \frac{1}{2}(3 + x - t) - \frac{1}{2}(1 + x + t)e^{-2t}$$