MAE/MSE 502 Fall 2022 HW1 Solution

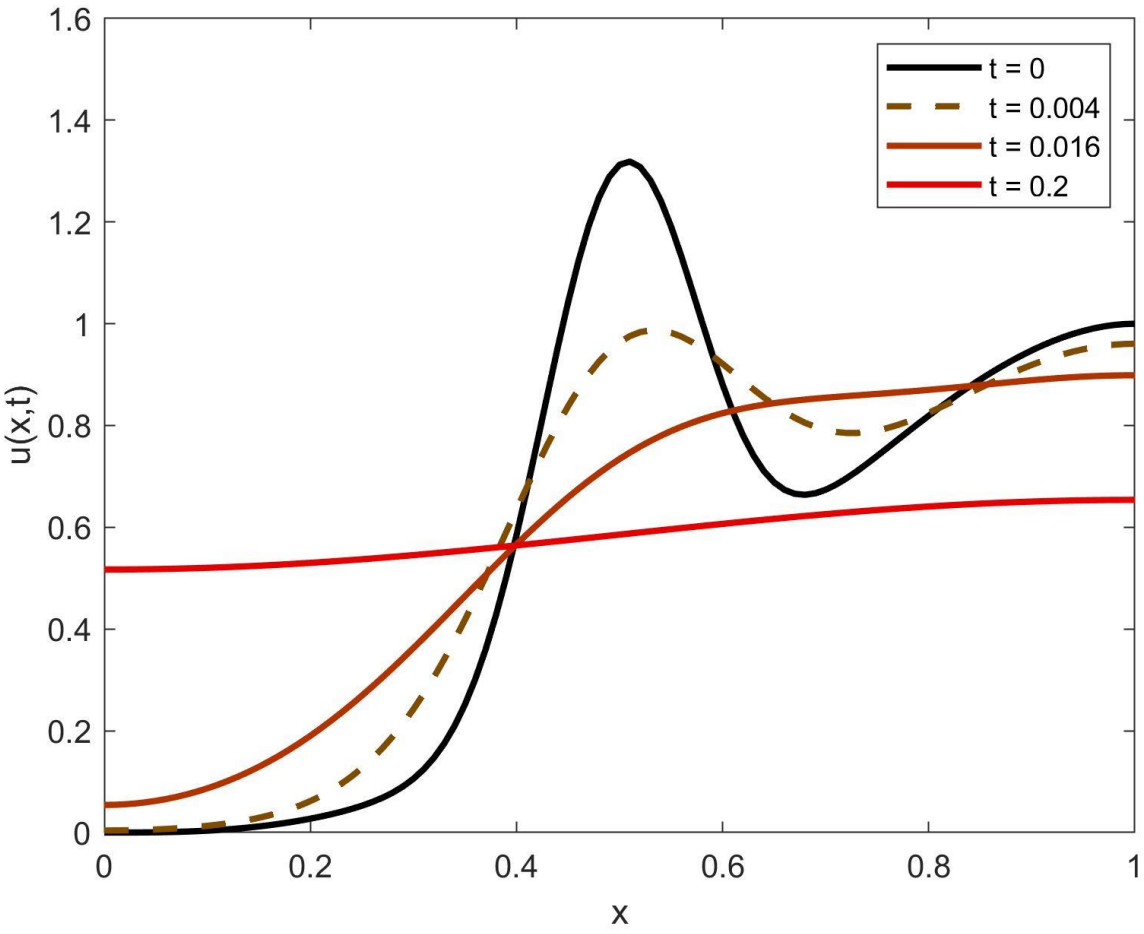
Problem 1(a)

$$u(x,t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x) \exp(-(n\pi)^2 t)$$

where

$a_0 = \int_0^1 P(x)\, dx$, and $a_n = 2\int_0^1 P(x) \cos(n\pi x) dx$ for $n > 0$.

Plot:



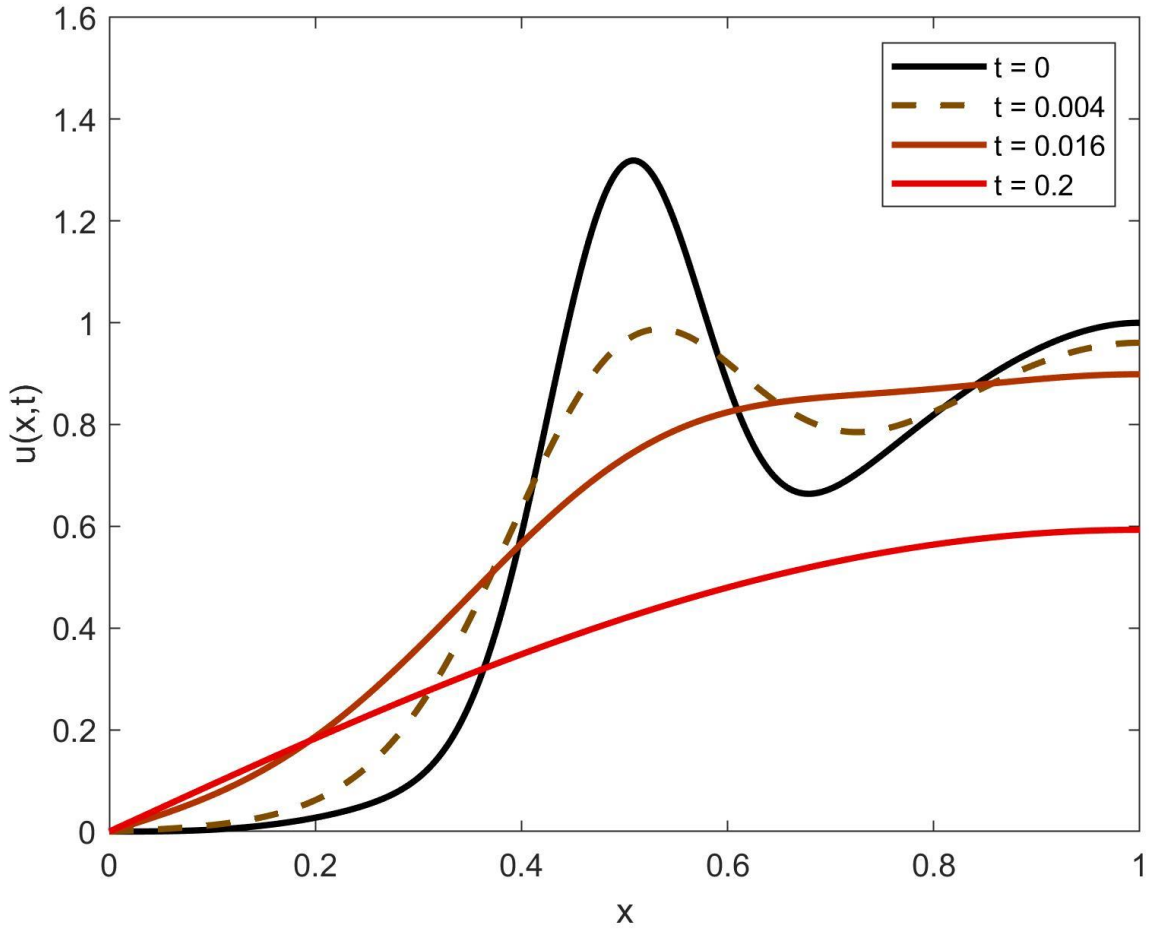
Problem 1(b)

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{2}\right) \exp\left(-\left(\frac{n\pi}{2}\right)^2 t\right)$$

where the summation is over odd values of n only, and

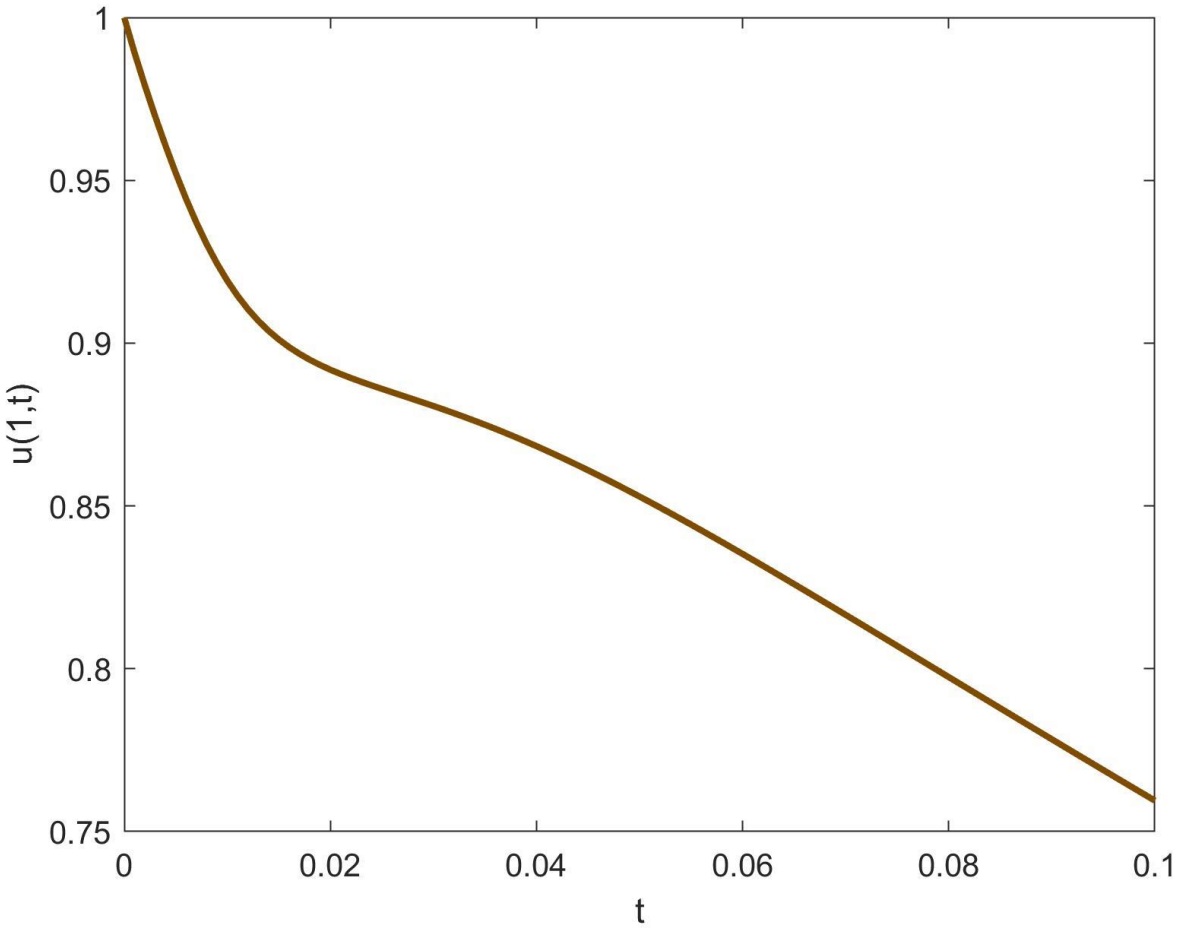
$a_n = 2\int_0^1 P(x)\,\sin(\frac{n\pi x}{2})dx$, for odd values of n.

Plot:



Problem 1(c)
$u(1,t)$ can be computed from the solution for Part (b) by setting *x* to 1.

Plot:



Problem 2

$$u(x,t) = \sin(\pi x)e^{3\pi^2(1-e^{-t})} + \sin(2\pi x)$$

Steady solution is

$$u_s(x) = \sin(\pi x)e^{3\pi^2} + \sin(2\pi x)$$

Problem 3

$$u(x,t) = e^{-t}\sqrt{1+2t} + e^{-t}\cos(x)$$

Problem 4

We will discuss the solution of this problem in class.