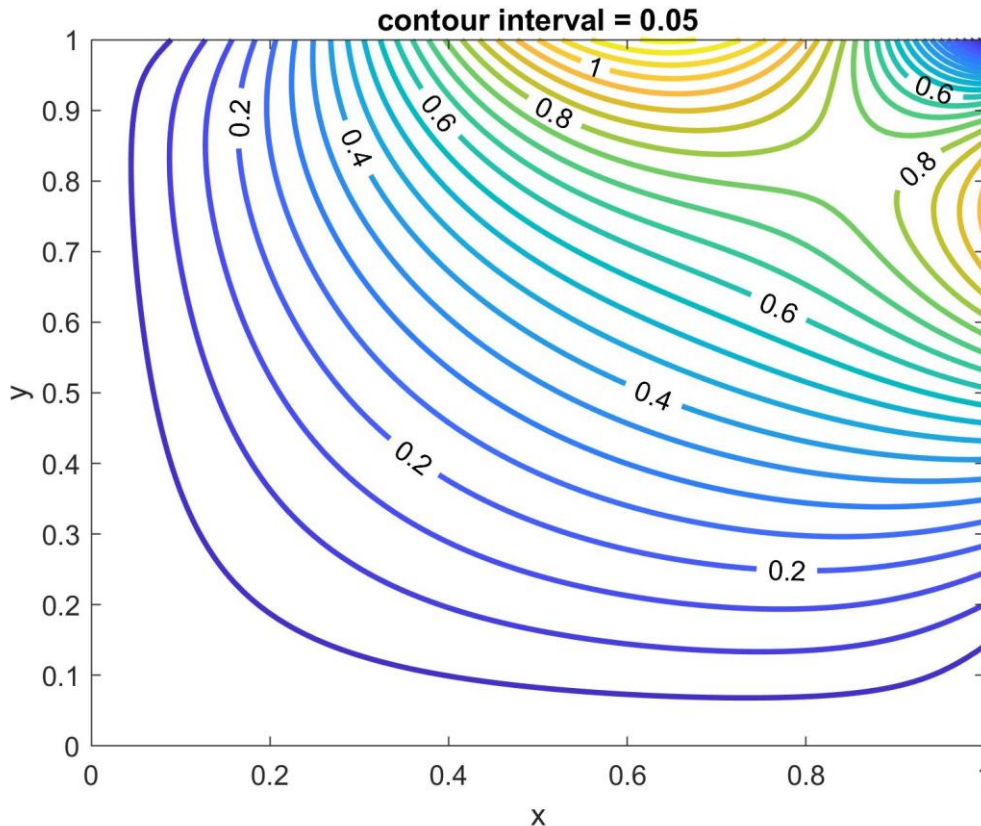


Problem 1

$$u(x, y) = \sum_{n=1}^{\infty} a_n \sin(n\pi y) \sinh(n\pi x) + b_n \sin(n\pi x) \sinh(n\pi y)$$

where

$$a_n = \frac{2}{\sinh(n\pi)} \int_0^1 2.5 (y^2 - y^6) \sin(\pi y) \sin(n\pi y) dy, \quad b_n = \frac{2}{\sinh(n\pi)} \int_0^1 2x \sin(\pi x) \sin(n\pi x) dx$$



Problem 2

System has infinite many solutions:

$$u(x, y) = x + C + \frac{\cosh(2\pi x) \cos(8\pi y)}{2\pi \sinh(2\pi)}, \quad C \text{ an arbitrary constant}$$

Problem 3

$$u(x, y) = \frac{\sin(4x)}{\sin(4)} + x \cos(4y) + \frac{\sinh(3x)}{\sinh(3)} \cos(5y)$$

Problem 4

$$u(x, y) = \left(2y + \frac{1}{y^4}\right) \sin(4x)$$