

MAE/MSE 502, Fall 2022 HW3 Solution

Problem 1

$$u(x, t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{100}\right) \cos\left(\frac{n\pi t}{100}\right)$$

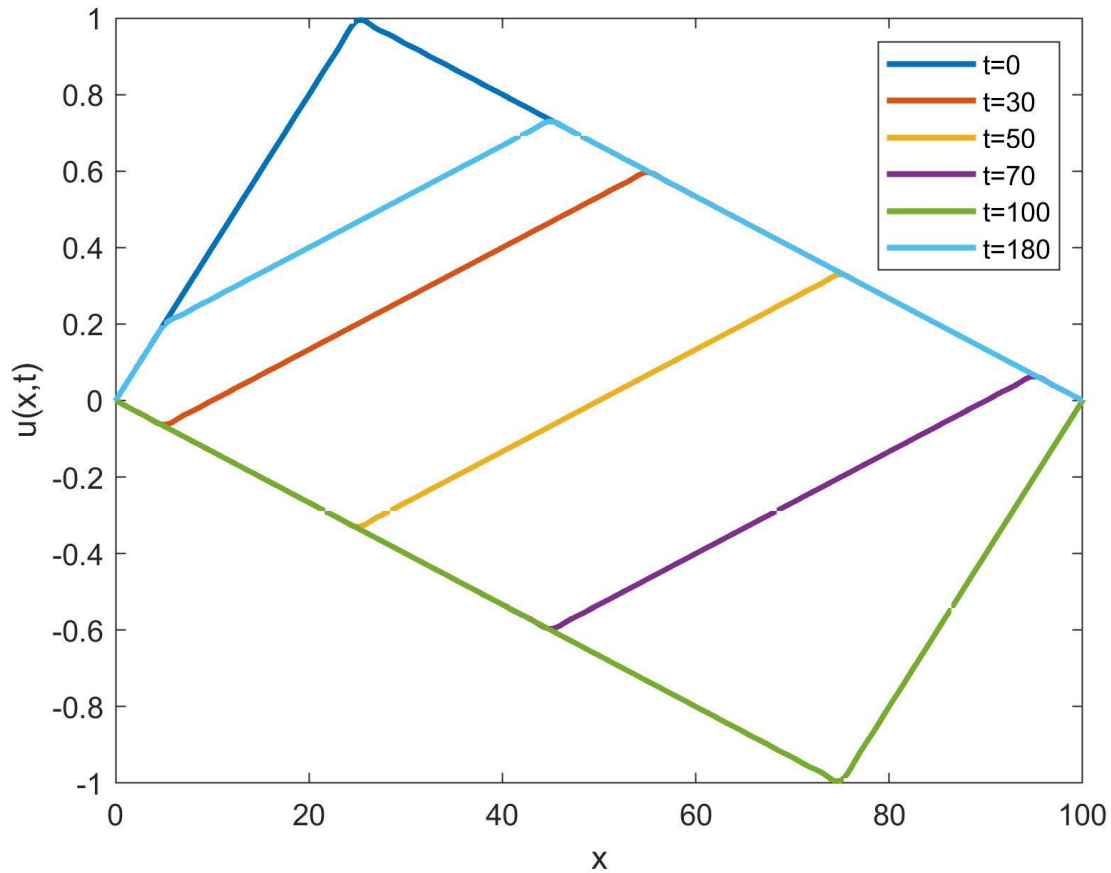
For Part (a):

$$a_n = \frac{1}{50} \left[ \int_0^{25} \frac{x}{25} \sin\left(\frac{n\pi x}{100}\right) dx + \int_{25}^{100} \frac{(100-x)}{75} \sin\left(\frac{n\pi x}{100}\right) dx \right].$$

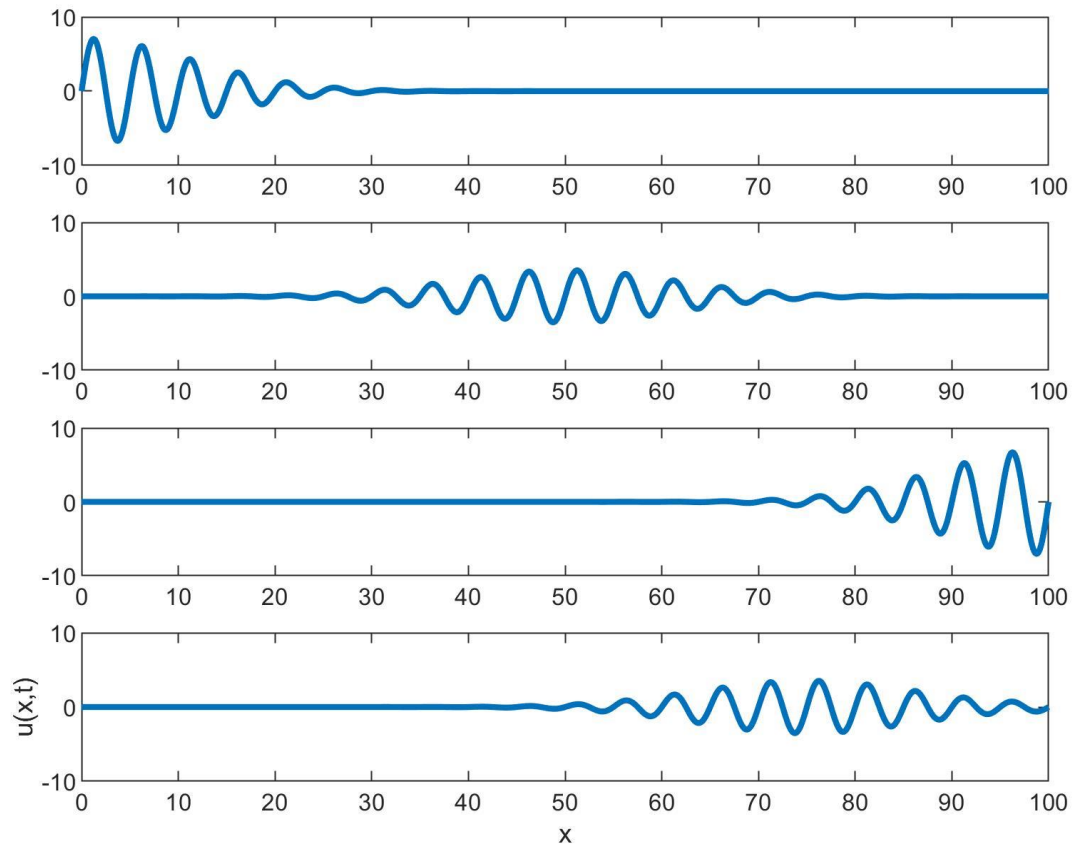
For Part (b):

$$a_n = \exp\left[-\left(\frac{n-40}{4}\right)^2\right], \text{ for } 30 \leq n \leq 50, \text{ and } a_n = 0 \text{ otherwise}$$

Plot for (a)



Plot for (b): Top to bottom:  $t = 0, 50, 100,$  and  $125$



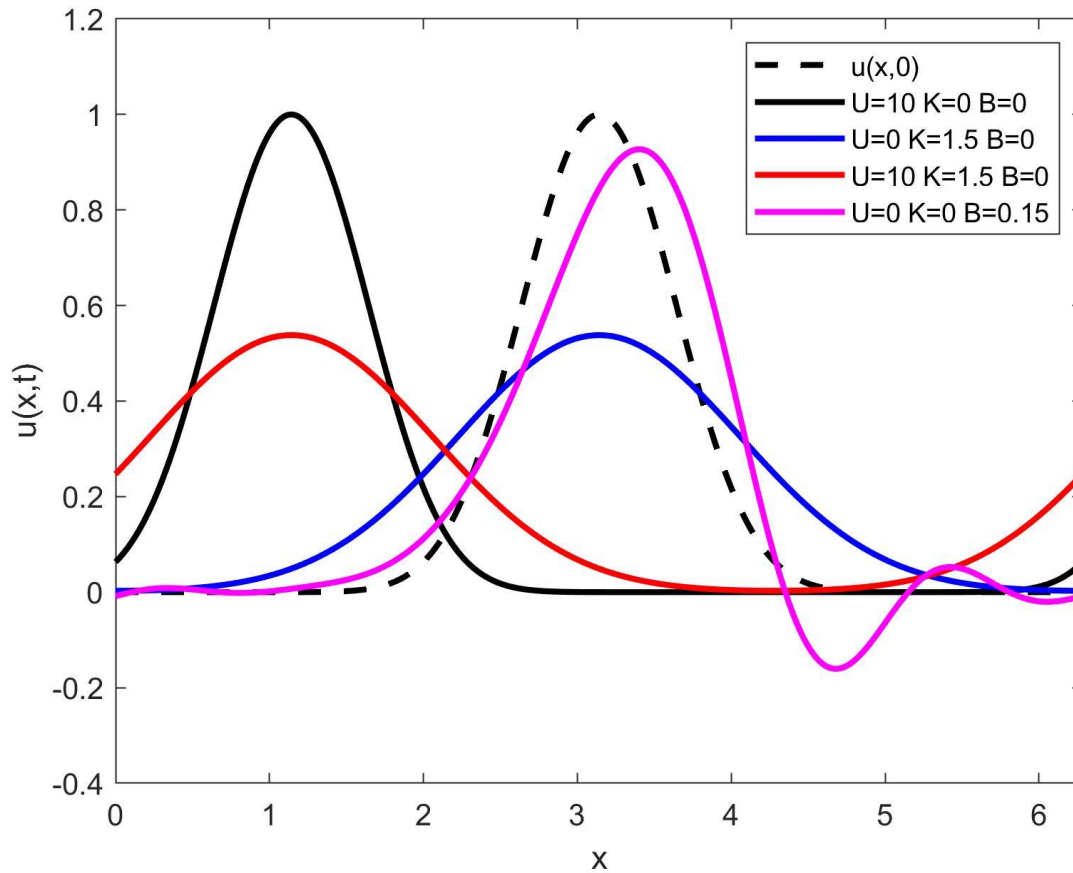
Problem 2

$$u(x, t) = \sum_{n=-\infty}^{\infty} C_n(0) e^{(inU - n^2K - in^3B)t + inx}$$

where

$$C_n(0) = \frac{1}{2\pi} \int_0^{2\pi} u(x, 0) e^{-inx} dx$$

Plot for Problem 2:



Problem 3

$$u(x, t) = e^{t^3} [\cos(x) + \sin(x)] + \cos(2x + 12t^2)$$

Problem 4

$$u(x, t) = \sin(x) \cosh(t) + \frac{1}{2} \cos(2x) \sinh(2t)$$

Problem 5

$$u(x, t) = e^{4t} + 2e^{-t} + \cos(x) + \frac{1}{10} [e^{3t} (3 \cos(x+t) + \sin(x+t)) - (3 \cos(x) + \sin(x))]$$