MAE/MSE 502, Fall 2022 Homework #4

A statement of collaboration is required. For Q2, Q3, and Q4 in this homework, we expect an exact solution expressed in only a finite number of terms and without any unevaluated integrals. The solution, $u(x,t)$, should be expressed in real functions of x and t and real numbers. There will be a deduction if these requirements are not satisfied.

Problem 1 (2.5 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 5$ and $t \ge 0$, solve the PDE,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},$$

with the boundary conditions (u_x denotes $\partial u/\partial x$),

(i) $u_x(0,t) = 3$
(ii) $u(5,t) = 16$
(iii) $u(x,0) = x^2 + 3x - 24$.

Plot the solution $u(x, t)$ as a function of x at $t = 0$, 3, and 10, and the steady solution. Please collect all 4 curves in the same plot. [Note: This is Q1 from midterm, except that in the exam you were only required to obtain the steady solution. For this problem, you may express the full solution as an infinite series, and truncate it when performing the numerical calculation to make the plot (in the same manner as HW1-Q1).]

Problem 2 (2.5 points)
For $u(x,t)$ defined on the domain of $0 \le x \le \pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} + u = t + e^{-t}\cos(x)$$

with the boundary conditions

(i) $u_x(0,t) = 0$ (ii) $u_x(\pi,t) = 0$ (iii) $u(x,0) = 0$.

Problem 3 (2 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 2 + \pi^2\cos(\pi x)$$

with the boundary conditions,

(i) $u(0,t) = 1$ (ii) $u(1,t) = -1$ (iii) $u(x,0) = \cos(\pi x) + \sin(\pi x) + x - x^2$.

Problem 4 (3 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial t \partial x^2} - \frac{\partial^2 u}{\partial x^2} = e^{-t} + \cos(x)$$

with periodic boundary conditions in x-direction, and the boundary conditions in t-direction given as

(i) $u(x,0) = 3$ (ii) $u_t(x,0) = 1$