

MAE/MSE 502, Fall 2022 Homework #5

A statement of collaboration is required. For all problems in this homework, we expect a closed-form solution without any unevaluated integrals. The solution, $u(x, t)$, must be written explicitly as a function of x and t .

Problem 1 (2.5 points) For $u(x, t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$e^{-t} \frac{\partial u}{\partial t} - 2x e^{-t} \frac{\partial u}{\partial x} = x u e^t + e^{-t}$$

with the boundary condition,

$$u(x, 0) = 1.$$

Plot the solution, $u(x, t)$, as a function of x at $t = 0, 0.1, 0.2$, and 0.3 . The plot should be made over the range of $1 \leq x \leq 5$. Please collect all four curves in a single plot.

Problem 2 (2.5 points) For $u(x, t)$ defined on the infinite domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\frac{\partial u}{\partial t} + 2u \frac{\partial u}{\partial x} = 2x + e^{2t}$$

with the boundary condition,

$$u(x, 0) = -x.$$

Problem 3 (3 points) For $u(x, t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\frac{\partial u}{\partial t} - u \frac{\partial u}{\partial x} = 1$$

with the boundary condition,

$$u(x, 0) = \begin{cases} 1, & \text{if } x < 1 \\ \frac{1}{x}, & \text{if } x \geq 1 \end{cases}$$

Plot the solution, $u(x, t)$, as a function of x at $t = 0, 0.2$ and 0.5 . The plot should be made over the range of $-1 \leq x \leq 3$. Please collect all three curves in a single plot.

Problem 4 (3 points) For $u(x, t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + 0.5x$$

with the boundary conditions in t -direction given as

$$(i) u(x, 0) = x \quad (ii) u_t(x, 0) = 0.$$