MAE/MSE 502, Spring 2022 Homework #3

Please follow the rules on collaboration as given in the document for Homework #1. Your work should include a statement of collaboration.

Problem 1 (2.5 points)
For $u(x, t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, consider the 1-D Wave equation,

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \ ,$$

with the boundary conditions,

(i) $u(0, t) = 0$, (ii) $u(1, t) = 0$, (iii) $u(x, 0) = P(x)$, (iv) $u_t(x, 0) = 0$.

(a) Solve the system with P(x) given as

$$P(x) = \begin{cases} 12\,x\,, & if\ 0 \le x \le 0.25 \\ 4 - 4x\,, & if\ 0.25 \le x \le 1 \end{cases}$$

and plot the solution as a function of x at $t = 0, 0.3, 0.5, 0.7, 1.0, 1.8$. Please collect all 6 curves in one plot. (See relevant remarks below HW1-Prob1 on the truncation of infinite series.)

(b) Solve the system with $P(x)$ given as (this emulates a "wave packet")

$$P(x) = \sum_{n=32}^{48} \exp\left[-\left(\frac{n-40}{5}\right)^2\right] \sin(n\pi x)\,,$$

and plot the solution as a function of x at $t = 0, 0.5, 1.0$, and 1.5. For this part, it is recommended that the four curves be plotted separately, for example arranged in 4 panels using "subplot" in Matlab.

Problem 2 (3 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, consider the PDE (in which U, K, and B are constants)

$$\frac{\partial u}{\partial t} = U\frac{\partial u}{\partial x} + K\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^3 u}{\partial x^3}$$

with periodic boundary conditions in the x-direction. The boundary conditions in the t-direction are given as

$$u(x,0) = \frac{[1-\cos(x)]^{10}}{1024}\ .$$

Solve the PDE by Fourier series expansion. Plot the solution $u(x, t)$ at $t = 0.3$ for the four cases: **(i)** ($U = 7$, $K = 0$, $B = 0$), **(ii)** ($U = 0$, $K = 0.3$, $B = 0$), **(iii)** ($U = 7$, $K = 0.3$, $B = 0$), and (iv) ($U = 0$, $K = 0$, $B = 0.1$), Also, plot the solution at $t = 0$ (which is the same for all four cases). Please collect all five curves in one plot.

For this problem, the solution can be expressed as an infinite series. To compute the values of u for the plot, the series needs to be truncated to a finite number of terms (see relevant remarks below HW1-Prob1). Matlab can be used to evaluate the expansion coefficients by numerical integration.

Problem 3 (2.5 points)
For $u(x, t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE,

$$\frac{\partial u}{\partial t} = t\frac{\partial^5 u}{\partial x^5} + 4t\frac{\partial^3 u}{\partial x^3} + \frac{\partial^2 u}{\partial x^2} + u \ ,$$

with periodic boundary conditions in the x-direction. The boundary condition in the t-direction is given as

$u(x, 0) = 1 + \sin(x) + \cos(2x)$.

We expect a closed-form solution which consists of only a finite number of terms and no unevaluated integrals. The solution should be expressed in real numbers and functions. Expect a deduction if an imaginary number "i" ($=\sqrt{-1}$) is left in the solution.

Problem 4 (2.5 points)
For $u(x, t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE,

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^4 u}{\partial x^4} + 2\frac{\partial^2 u}{\partial x^2} + u \ ,$$

with periodic boundary conditions in the x-direction. The boundary conditions in the t-direction are given as

(i) $u(x, 0) = \cos(x)$

(ii) $u_t(x, 0) = 1 + \sin(x)$.

We expect a closed-form solution which consists of only a finite number of terms and no unevaluated integrals. The solution should be expressed in real numbers and functions. Expect a deduction if an imaginary number "i" ($=\sqrt{-1}$) is left in the solution.

One more question in net page!

Problem 5 (2.5 points)
For $u(x,t)$ defined on the domain of $0 \leq x \leq 2\pi$ and $t \geq 0$, solve the PDE

$$\frac{\partial^2 u}{\partial t^2} - 2\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} + u = 0\,,$$

with periodic boundary conditions in the x-direction, and the boundary conditions at $t = 0$ given as

(i) $u(x,0) = \cos(x)$

(ii) $u_t(x,0) = \sin(2x)$.

We expect a closed-form solution which consists of only a finite number of terms and no unevaluated integrals. The solution should be expressed in real numbers and functions. Expect a deduction if an imaginary number "i" (=$\sqrt{-1}$) is left in the solution.