MAE/MSE 502, Spring 2022   Homework #4

Please include a statement of collaboration in your work.

For all problems in this homework, we expect an exact solution expressed in only a finite number of terms and without any unevaluated integrals. The solution, $u(x,t)$, should be expressed in real functions of x and t and real numbers. There will be a deduction if these requirements are not satisfied.

Problem 1 (3 points)
For $u(x,t)$ defined on the domain of $0 \le x \le \pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u + e^t + \cos(x)\sin(t)$$

with the boundary conditions

(i) $u_x(0,t) = 0$ (ii) $u_x(\pi,t) = 0$ (iii) $u(x,0) = 1 + \cos(3x)$.

Problem 2 (3 points)
For $u(x,t)$ defined on the domain of $0 \le x \le \pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - \sin(x)$$

with the boundary conditions

(i) $u(0,t) = 0$ (ii) $u(\pi,t) = 0$ (iii) $u(x,0) = \sin(x)$ (iv) $u_t(x,0) = \sin(x)$

Problem 3 (3 points)
For $u(x,t)$ defined on the domain of $0 \le x \le \pi/2$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u + 1$$

with the boundary conditions,

(i) $u(0,t) = -1$ (ii) $u(\pi/2\,,t) = 0$ (iii) $u(x,0) = \sin(x) + \sin(2x) - 1$.

Problem 4 (3 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^3 u}{\partial x^3} + \sin(t)$$

with periodic boundary conditions in x-direction, and the boundary conditions in t-direction given as

(i) $u(x,0) = \cos(x)$ (ii) $u_t(x,0) = 1$