MAE/MSE 502, Spring 2022 Homework #5

For all problems in this homework, we expect a closed-form solution without any unevaluated integrals. The solution, $u(x, t)$, must be written explicitly as a function of x and t.

Problem 1 (2.5 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$e^t \frac{\partial u}{\partial t} + x\, e^t \frac{\partial u}{\partial x} = x\, u + 1$$

with the boundary condition,

$$u(x, 0) = 0.$$

Make a plot of the solution, $u(x,t)$, as a function of x at $t = 0.1$, 0.3, and 0.5. Please collect all 3 curves in one plot.

Problem 2 (2.5 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 2$$

with the boundary condition,

$$u(x, 0) = \begin{cases} -\frac{1}{x}\,, & if\ x \leq -1 \\ \\ 1\,, & if\ x > -1 \end{cases}$$

Plot the solution, $u(x, t)$, as a function of x at $t = 0$, 0.2 and 0.5. The recommended range for plotting is $-3 \leq x \leq 1$. Please collect all three curves in a single plot.

Problem 3 (2.5 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = x + 1$$

with the boundary condition,

$$u(x, 0) = -x\ .$$

Problem 4 (2.5 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + x$$

with the boundary conditions in t-direction given as

(i) $u(x, 0) = 1$ (ii) $u_t(x, 0) = 0$.