MAE/MSE 502, Spring 2023 HW2 Solution

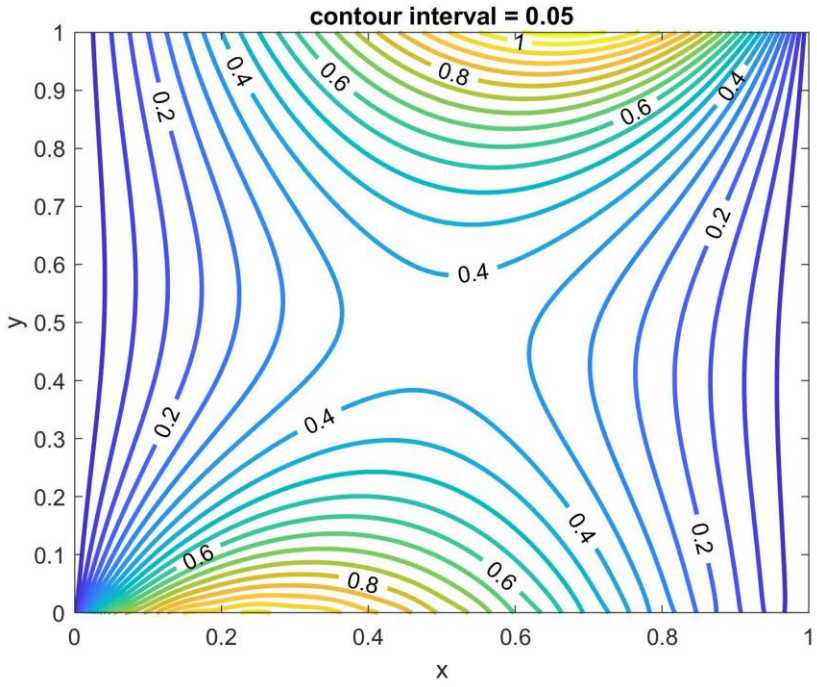
Problem 1

[Note: The solution is essentially that given in Lecture 11. We write it in a slightly more compact way.]

$$u(x,y) = \sum_{n=1}^{\infty} [a_n \sinh(n\pi y) + b_n \sinh(n\pi(1-y))] \sin(n\pi x)$$

where

$$a_n = \frac{2}{\sinh(n\pi)} \int_0^1 2\,(x - x^5) \sin(n\pi x)\, dx \;,\; b_n = \frac{2}{\sinh(n\pi)} \int_0^1 \sin\left(\pi\sqrt{x}\right) \sin(n\pi x) dx$$



Problem 2

(a) Solvability condition is satisfied. (b) The system has infinite many solutions:

$$u(x,y) = x + C + \frac{\cosh(4\pi x)\cos(3\pi y)}{4\pi \sinh(4\pi)} \;, C \text{ an arbitrary constant}$$

Problem 3

$$u(x,y) = (5e^{y} - e^{3y}) \sin(2x)$$

Problem 4

[Note: One could just define $\hat{u} \equiv u - 3$, then the system for $\hat{u}$ is identical to that solved in Lecture 10.]

$$u(x,y) = 3 + \frac{\sin(2023\pi y)\sinh(2023\pi x)}{\sinh(2023\pi)}$$