

MAE/MSE 502, Spring 2023 HW3 Solution

Problem 1

$$u(x, t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{5}\right) \cos\left(\frac{n\pi t}{5}\right)$$

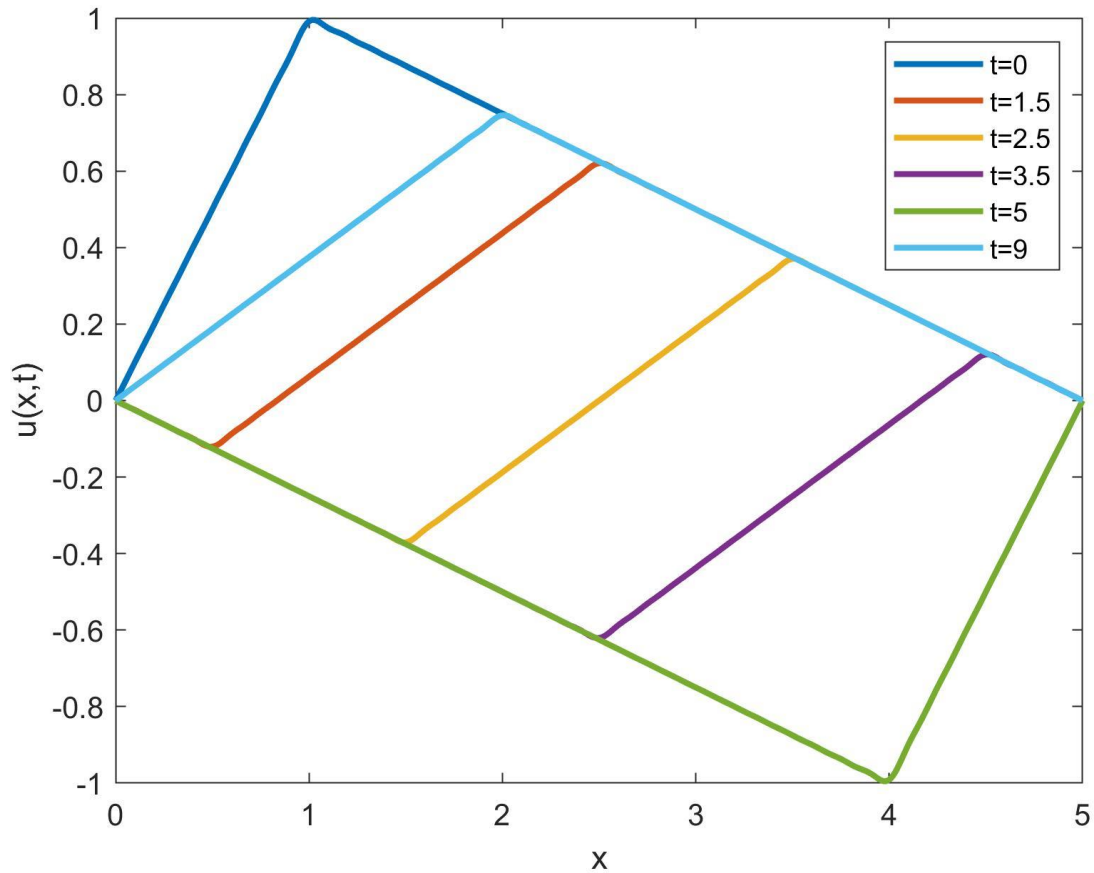
For Part (a):

$$a_n = \frac{2}{5} \left[\int_0^1 x \sin\left(\frac{n\pi x}{5}\right) dx + \int_1^5 (1.25 - 0.25x) \sin\left(\frac{n\pi x}{5}\right) dx \right].$$

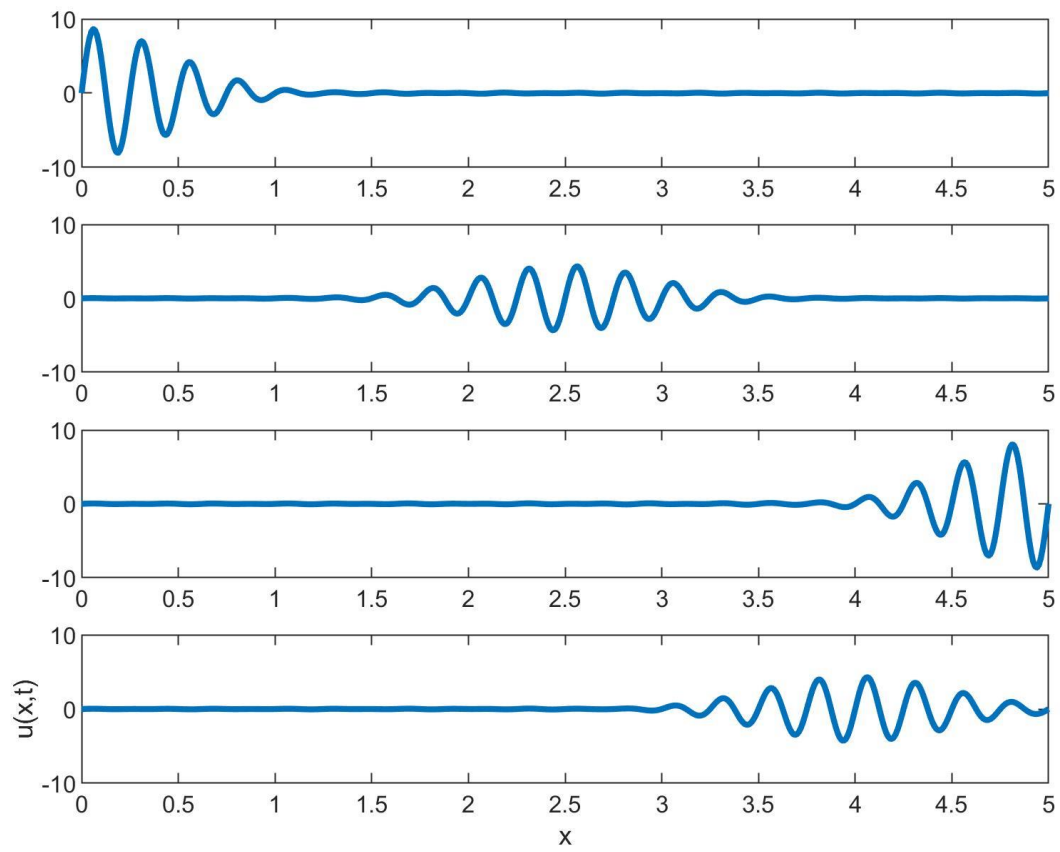
For Part (b):

$$a_n = \exp\left[-\left(\frac{n-40}{5}\right)^2\right], \text{ for } 32 \leq n \leq 48, \text{ and } a_n = 0 \text{ otherwise}$$

Plot for (a)



Plot for (b): Top to bottom: $t = 0, 2.5, 5,$ and 6



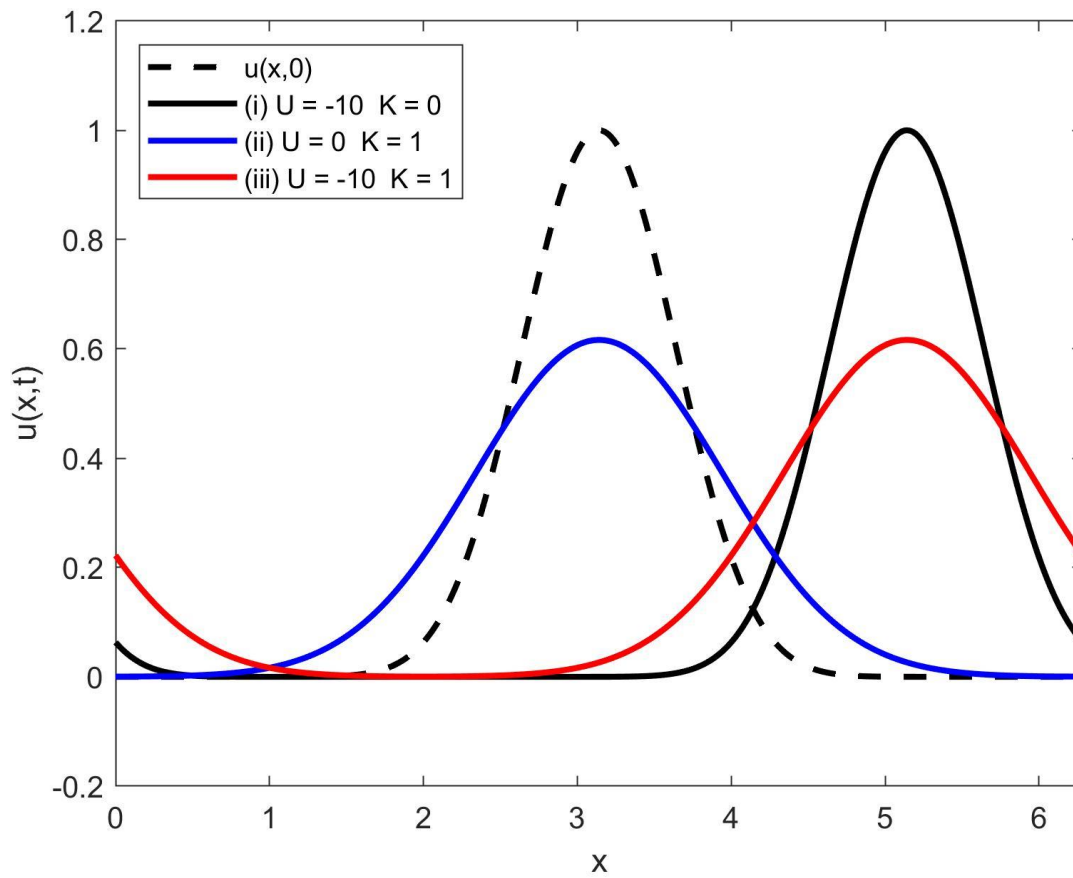
Problem 2

$$u(x, t) = \sum_{n=-\infty}^{\infty} C_n(0) e^{(inU - n^2K - in^3B + in^5D)t + inx}$$

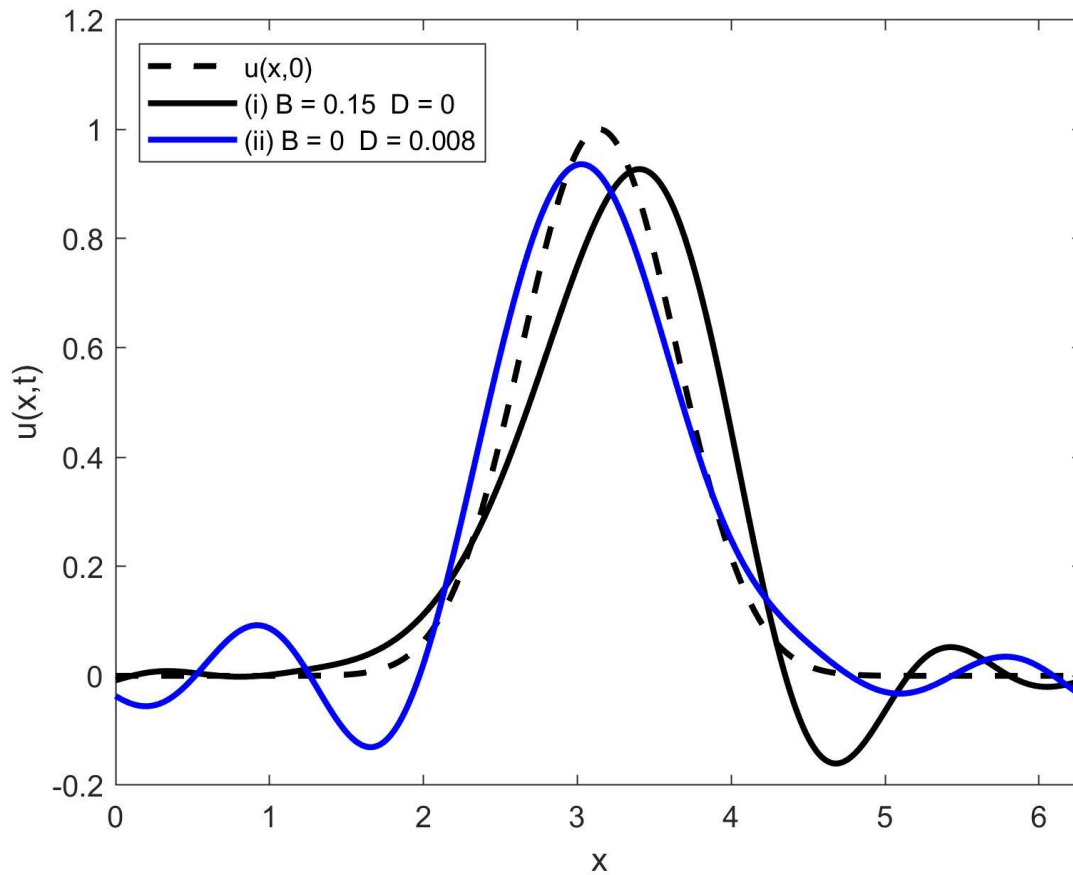
where

$$C_n(0) = \frac{1}{2\pi} \int_0^{2\pi} u(x, 0) e^{-inx} dx$$

Plot for 2(a):



Plot for 2(b):



Problem 3

$$u(x, t) = e^{-t} + \sin(x - 4t^2) + e^{80t} \cos(3x)$$

Problem 4

$$u(x, t) = \cosh(t) + \sin(x + t) - \sin(x)$$

Problem 5

$$u(x, t) = \sin(x) + \frac{1}{8} [e^{5t} - e^{-3t}] \cos(2x)$$