MAE/MSE 502, Spring 2023 Homework #4

A statement of collaboration is required. For Q1, Q2, and Q4, we expect an exact solution expressed in only a finite number of terms and without any unevaluated integrals. The solution, $u(x,t)$, should be expressed in real functions of x and t and real numbers. There will be a deduction if these requirements are not satisfied.

Problem 1 (2 points)
For $u(x,t)$ defined on the domain of $0 \le x \le \pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2} + u + e^{-2t}\sin(x)$$

with the boundary conditions

(i) $u(0,t) = 0$ (ii) $u(\pi,t) = 0$ (iii) $u(x,0) = \sin(2x)$.

Problem 2 (2.5 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 2\pi$ and $t \ge 0$, solve the PDE

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^4 u}{\partial x^4} - u + \cos(x)\cos(t)$$

with periodic boundary conditions in x-direction, and the boundary conditions in t-direction given as

(i) $u(x,0) = \sin(x)$ (ii) $u_t(x,0) = 1$

Problem 3 (2.5 points)
Let us revisit HW1-Q4 but now take one step further to reach the full solution. For $u(x,t)$ defined on the domain of $0 \le x \le 5$ and $t \ge 0$, solve the PDE,

$$\frac{\partial u}{\partial t} = 100\frac{\partial^2 u}{\partial x^2} ,$$

with the boundary conditions (u_x denotes $\partial u/\partial x$),

(i) $u_x(0,t) = 3$ (ii) $u(5,t) = 16$ (iii) $u(x,0) = x^2 + 3x - 24$.

Plot the solution $u(x, t)$ as a function of x at $t = 0$, 0.05, and 0.15, and the steady solution. Collect all 4 curves in the same plot. [For this problem, you may express the full solution as an infinite series, and truncate it when performing the numerical calculation to make the plot (in the same manner as HW1-Q1).]

Problem 4 (3 points)
For $u(x,t)$ defined on the domain of $0 \le x \le 1$ and $t \ge 0$, solve the PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 4 - \pi^2\sin(\pi x)$$

with the boundary conditions,

(i) $u(0,t) = 1$ (ii) $u(1,t) = -1$ (iii) $u(x,0) = 1 - 2x^2$.

Plot the solution $u(x, t)$ as a function of x at $t = 0$, 0.03, and 0.12, and the steady solution. Collect all 4 curves in one plot.