MAE/MSE 502, Spring 2023 Homework #5

A statement of collaboration is required. For all problems, we expect a closed-form solution without any unevaluated integrals. The solution, $u(x, t)$, must be written explicitly as a function of x and t.

Problem 1 (2 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\frac{\partial u}{\partial t} + 3u\frac{\partial u}{\partial x} + e^{3t}\frac{\partial u}{\partial x} = 3x$$

with the boundary condition,

$$u(x,0) = 1 - x\ .$$

Problem 2 (3 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\frac{\partial u}{\partial t} + e^{u}\frac{\partial u}{\partial x} - t\ \frac{\partial u}{\partial x} = e^{-u}$$

with the boundary condition,

$$u(x,0) = \begin{cases} 0\,, & if\ x < 1 \\ ln(x)\,, & if\ x \geq 1 \end{cases}$$

where $ln(x)$ is the natural logarithm of x. (Note that $e^{ln(A)} = A$.) Plot the solution, $u(x, t)$, as a function of x at $t = 0$, 0.5, and 1. The plot should be made over the range of $0 \leq x \leq 5$. Collect all three curves in a single plot.

Problem 3 (2 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\frac{\partial u}{\partial t} + x\ \frac{\partial u}{\partial x} = x\ u\ e^{-t} + 1$$

with the boundary condition,

$$u(x,0) = 1.$$

Plot the solution, $u(x, t)$, as a function of x at $t = 0$, 0.2, 0.5, and 1.5. The plot should be made over the range of $1 \leq x \leq 5$. Collect all four curves in a single plot.

Problem 4 (3 points)
For $u(x,t)$ defined on the domain of $-\infty < x < \infty$ and $t \geq 0$, solve the PDE

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + x$$

with the boundary conditions in t-direction given as

(i) $u(x,0) = x$ (ii) $u_t(x,0) = 1$.