MAE 502 Partial Differential Equations in Engineering

Spring 2009 Mon/Wed 5:30-6:45 PM SCOB 201

- Instructor: Huei-Ping Huang (hp.huang@asu.edu), ISTB2 Room 219A
- Office hours:
  Tuesday 3:00-4:30 PM, Wednesday after class (nominally 6:45-8:00 PM), or by appointment
- Course website
  http://www.public.asu.edu/~hhuang38/MAE502.html

Course Outline

I. Analytic treatment for linear PDE

1. Overview of PDE

Commonly encountered PDEs in engineering and science
Types of PDEs, the physical phenomena they represent, and relevant boundary conditions

2. Some analytical solutions of PDEs

Separation of variables, Method of characteristics, etc.

3. Review of boundary value problems with ODE

Sturm-Liouville Problem and orthogonal functions;
Representation using orthogonal basis

4. Fourier Series

Solution of ODE and PDE by Fourier Series expansion

5. Fourier transform and Laplace transform

Solution of PDE by Fourier/Laplace transform

6. Series expansion and integral transform methods for PDEs with non-Cartesian geometry (if time permits)

(continued)

II. Numerical methods for PDE

7. Introduction to Numerical solution of PDE

Overview; Numerical error and stability condition
Evolution equations; Elliptic equations with closed boundary
Spectral method (if time permits)

III. Additional topics (if time permits)

8. Brief introduction to nonlinear PDE

Examples of nonlinear PDEs for real world phenomena; Behavior of their solutions; Conservation laws; Strategies for numerical solutions

9. Miscellanies

Green's function and applications to solutions of ODE and PDE
Asymptotic solutions

[1] Primary textbook: "*Applied Partial Differential Equations*", 4th Edition, by R. Haberman. Prentice Hall. **Required**

[2] Additional material for numerical methods will be drawn from "*Applied Partial Differential Equations*", by P. DuChateau and D. Zachmann. Dover Publications. **Recommended**

[3] Lecture notes by instructor

- Will follow **[1]** as closely as possible for Part I (analytic solutions) but some departure is expected
- Book **[2]** is better organized but with relatively terse treatments on analytic solutions - useful alternative to **[1]** if you want "second opinion".
- Book **[2]** has detailed treatments on numerical methods for PDE.

Grade:

50% Homework/projects
20% Midterm (1 exam)
30% Final

Useful things to review ...

- Basic material for ordinary differential equations (ODE), linear algebra, and calculus of multiple variables.

- Working knowledge of Matlab (or an alternative with similar programming/graphics capability) - useful for plotting results for homework/project and/or speeding up some calculations for homework.
  This is not an absolute necessity but might give you a slight edge.

Access to Matlab: https://apps.asu.edu

(i) Login using ASURITE ID/password

(ii) Choose appropriate version of Matlab for your computer. (Matlab 2007b works for most PC)

Use office hours wisely ...

Instructor will help you to

- Catch up with the class when you are falling behind
- Get an update of the latest happening when you miss a class
- Resolve technical difficulties related to homework/projects

Cannot make it to regular office hours? --> Schedule an appointment

A few quick examples of PDEs

(Not to worry about details - they will be discussed in later lectures)

Heat (or diffusion) equation

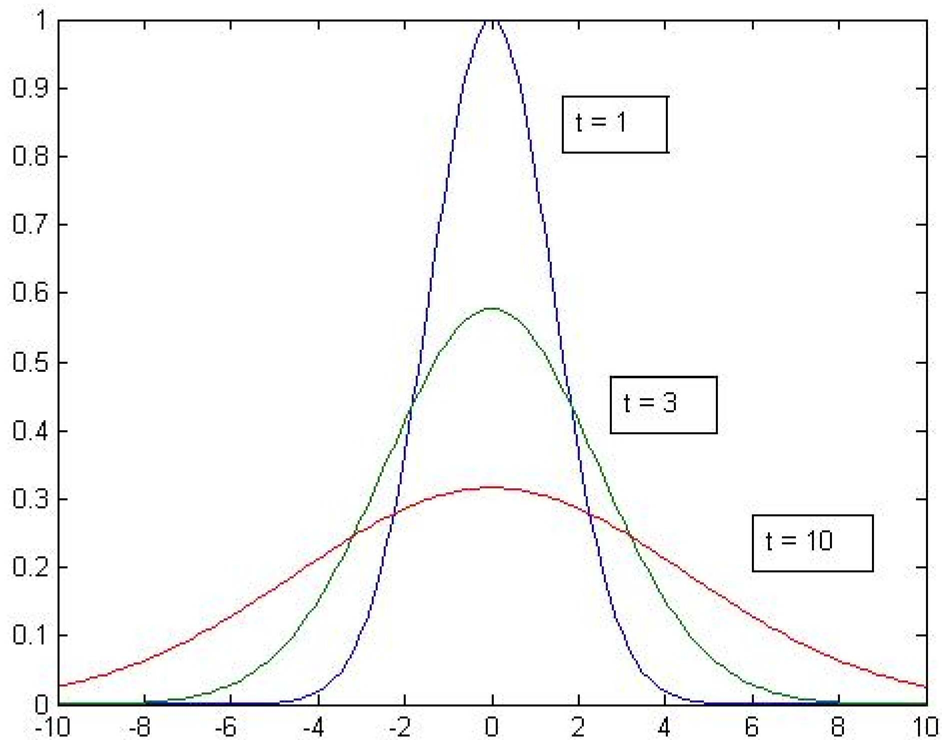
To help explain the correspondence between a PDE and a real world phenomenon, we will use *t* to denote time and (x, y, z) to denote the 3 spatial coordinates

Heat (or diffusion) equation: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, *describes the diffusion of temperature or the density of a chemical constituent from an initially concentrated distribution (e.g., a "hot spot" on a metal rod, or a speck of pollutant in the open air)*

A typical solution (when the initial distribution of *u* is a "spike"): $u(x,t) = \frac{1}{\sqrt{t}} \exp(-\frac{x^2}{4t})$

(Exercise: Verify that this solution does satisfy the original equation)

The figure in next page shows this solution at a few different times. As *t* increases, $u(x, t)$ becomes broader; Its maximum decreases but its "center of mass" does not move. These features characterize a "diffusion process".

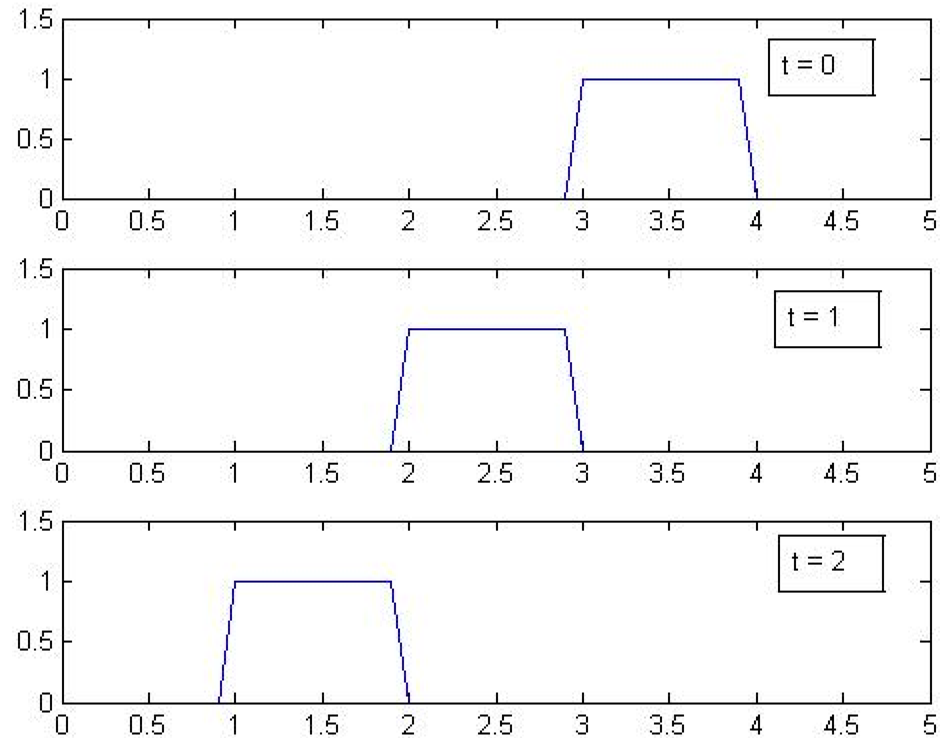


Solution of the heat equation at different times. The three curves are $u(x, 1)$, $u(x, 3)$, and $u(x,10)$

Linear advection (transport) equation

Linear advection equation: $\frac{\partial u}{\partial t} = c\,\frac{\partial u}{\partial x}$ *, describes the constant movement of an initial distribution of u with a "speed" of $-c$ along the x-axis. The distribution moves while preserving its shape.*

A typical solution: $u(x, t) = F(\xi)$, $\xi \equiv x + \mathrm{c}t$; F can be any function that depends only on $x+\mathrm{c}t$.
(Exercise: Verify that this is indeed a solution of the original equation.)
The following figure illustrate the behavior of the solution with c = 1. The initial condition, $u(x, t = 0)$, is a "top hat" structure. At later times, this structure moves to the left with a "speed" of $\delta x/\delta t = -1$ while preserving its shape. (The δx and δt here are the increments in space and time in the figure.)

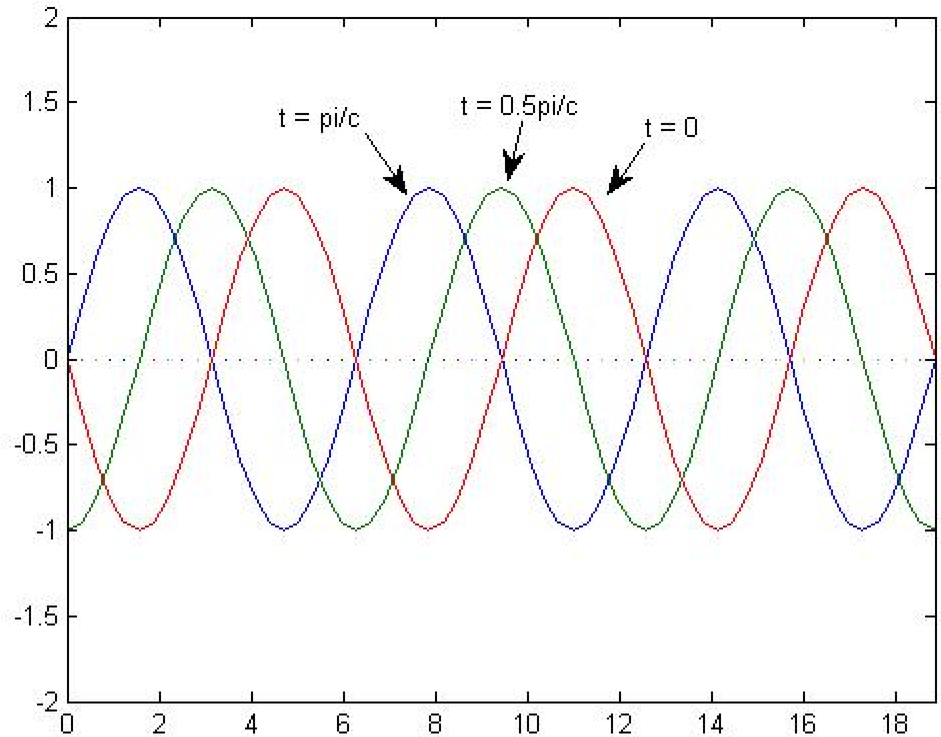


The 3 panels are $u(x, 0)$, $u(x, 1)$, and $u(x, 2)$

Linear wave equation

Linear wave equation: $\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$, *describes wave motion*

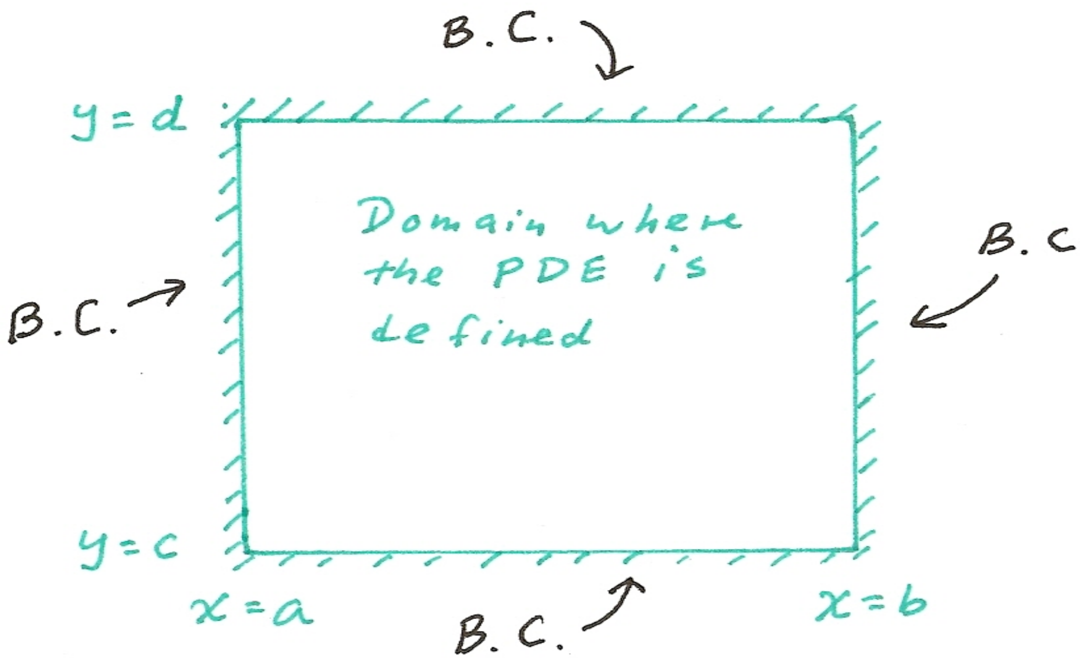
For example, a simple traveling sinusoidal structure, $u(x, t) = sin(x + \mathrm{c}t)$, as illustrated in the next figure, is a solution of the equation. (While at this level the solution is similar to that of the linear advection equation, more interesting behavior would emerge when we consider the superposition of different sinusoidal "modes", and when we introduce more interesting boundary conditions for the two equations.)


t = pi/c
t = 0.5pi/c
t = 0
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
2
4
6
8
10
12
14
16
18

Laplace equation

Laplace equation: $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$. (It belongs to the more general class of *elliptic equations*.)

It is usually defined on a closed domain as illustrated in the following. In this case, boundary conditions need to be specified at all of the four "walls" while we seek the solution within the closed domain that satisfies both the PDE and the boundary conditions.



Heat equation in two and three spatial dimensions

Heat equation in two- and three-dimensions:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \qquad \text{(2-D)}$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \qquad \text{(3-D)}$$

The behavior of the solutions of these equations is similar to that of the 1-D heat equation. An initially concentrated distribution in u will spread in space and become more smooth as t increases.