

## MAE502 Homework #2

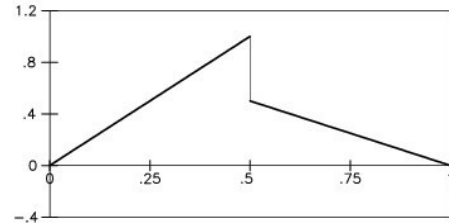
### Prob. 1 (4 points)

(a) Work out the Fourier Sine series expansion,

$$F(x) = \sum_{n=1}^{\infty} a_n \sin(n\pi x) ,$$

for the following function defined on  $x \in [0,1]$ ,

$$\begin{aligned} F(x) &= 2x , & 0 \leq x \leq 1/2 \\ &= 1-x , & 1/2 < x \leq 1 . \end{aligned}$$



A sketch of  $F(x)$  is shown at right; Notice a discontinuity at  $x = 1/2$ .

(b) Plot the original  $F(x)$  and its Fourier Sine series representation truncated at  $n = 5, 10,$  and  $30$ . Please collect all four curves in a single plot. What are the values of  $F(x)$  at  $x = 0.35$  for the three cases truncated at  $n = 5, 10,$  and  $30$  using Fourier Sine series expansion? Compare them to the exact value,  $F(0.35)$ , to determine the percentage error (using the exact value as denominator) for the three cases. Repeat the exercise for  $x = 0.49$  (a point close to the discontinuity). Discuss the results.

(c) Find the analytic expression for the value of  $F(x)$  at  $x = 1/2$  using the Fourier Sine series representation in (a). Define  $S(N)$  as the value of  $F(1/2)$  calculated from the Fourier Sine series truncated at  $n = N$ , plot  $S(N)$  as a function of  $N$  for the range  $1 \leq N \leq 30$ . What value does  $S(N)$  approach to at large  $N$ ?

### Prob 2. (5 points)

(a) Solve the Laplace equation for  $u(x, y), x \in [0,1], y \in [0,1]$ ,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 ,$$

with the boundary conditions,

$$(I) u(x, 0) = 0, \quad (II) u(x, 1) = P(x), \quad (III) u(0, y) = F(y), \quad (IV) u(1, y) = 0 ,$$

where

$$P(x) = \sin^2(\pi x) \quad (\text{Note that it is "sine square", not just "sine".}) ,$$

$$\begin{aligned} F(y) &= y , & 0 \leq y \leq 1/2 \\ &= 1-y , & 1/2 < y \leq 1 . \end{aligned}$$

(b) Plot the solution,  $u(x, y)$ , as a color/contour map for  $x \in [0,1], y \in [0,1]$  in the same fashion as the color/contour map in Matlab Example #3. When doing so, truncate all of the infinite series that appear in your solution at  $n = 20$  (i.e., retain 20 terms in these series.) Indicate the contour levels in your plot. (The recommended contour interval is 0.05, with min = 0.05 and max = 0.95.) Discuss the result. With the 20-term truncation, what are the values of  $u(x, y)$  at  $(x, y) = (0.25, 0.25)$  and  $(x, y) = (0.5, 0.75)$ ?

*If you do not have a color printer, a contour-only (no color shading) map is also acceptable for 2(b).*