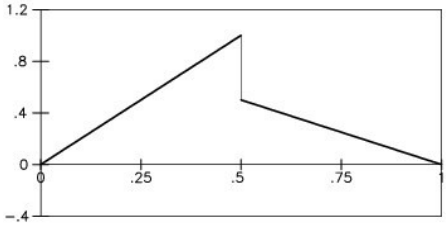
MAE502 Homework #2

Prob. 1 (**4 points**)
(a) Work out the Fourier Sine series expansion,

$$F(x) = \sum_{n=1}^{\infty} a_n \sin(n\pi x) \quad ,$$



for the following function defined on $x \in [0,1]$,

$$\begin{aligned} F(x) &= 2x, \quad 0 \le x \le 1/2 \\ &= 1-x, \quad 1/2 < x \le 1 . \end{aligned}$$

A sketch of $F(x)$ is shown at right; Notice a discontinuity at $x = 1/2$.

(b) Plot the original $F(x)$ and its Fourier Sine series representation truncated at $n = 5$, 10, and 30. Please collect all four curves in a single plot. What are the values of $F(x)$ at $x = 0.35$ for the three cases truncated at $n = 5$, 10, and 30 using Fourier Sine series expansion? Compare them to the exact value, $F(0.35)$, to determine the percentage error (using the exact value as denominator) for the three cases. Repeat the exercise for $x = 0.49$ (a point close to the discontinuity). Discuss the results.

(c) Find the analytic expression for the value of $F(x)$ at $x = 1/2$ using the Fourier Sine series representation in (a). Define $S(N)$ as the value of $F(1/2)$ calculated from the Fourier Sine series truncated at $n = N$, plot $S(N)$ as a function of N for the range $1 \le N \le 30$. What value does $S(N)$ approach to at large N?

Prob 2. (5 points)
(a) Solve the Laplace equation for $u(x, y)$, $x \in [0,1]$, $y \in [0,1]$,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad ,$$

with the boundary conditions,

(I) $u(x, 0) = 0$, (II) $u(x, 1) = P(x)$, (III) $u(0, y) = F(y)$, (IV) $u(1, y) = 0$,
where
$P(x) = \sin^2(\pi x)$ (Note that it is "sine square", not just "sine".) ,

$$\begin{aligned} F(y) &= y, \quad 0 \le y \le 1/2 \\ &= 1-y, \quad 1/2 < y \le 1 . \end{aligned}$$

(b) Plot the solution, $u(x, y)$, as a color/contour map for $x \in [0,1]$, $y \in [0,1]$ in the same fashion as the color/contour map in Matlab Example #3. When doing so, truncate all of the infinite series that appear in your solution at $n = 20$ (i.e., retain 20 terms in these series.) Indicate the contour levels in your plot. (The recommended contour interval is 0.05, with min = 0.05 and max = 0.95.) Discuss the result. With the 20-term truncation, what are the values of $u(x, y)$ at $(x, y) = (0.25, 0.25)$ and $(x, y) = (0.5, 0.75)$?

If you do not have a color printer, a contour-only (no color shading) map is also acceptable for 2(b).