

## MAE502 Homework #3

### Prob. 1 (6 points)

(a) Solve the two-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} ,$$

for  $u(x, y, t)$  defined on  $x \in [0,1]$ ,  $y \in [0,1]$ , and  $t \in [0, \infty)$ , with the following boundary conditions

(i)  $u(0, y, t) = 0$

(ii)  $u(1, y, t) = 0$

(iii)  $u(x, 0, t) = 0$

(iv)  $u(x, 1, t) = 0$

(v)  $u(x, y, 0) = \sin^2(\pi x)[\sin(\pi y) + \sin(2\pi y)]$

(vi)  $u_t(x, y, 0) = 0$  (Note:  $u_t \equiv \partial u / \partial t$ ).

(b) Plot the solution  $u(x, y, t)$  at  $t = 0$  (initial state), 0.3, 0.7, and 2.47 as contour/color maps in the same fashion as Matlab Example #3 or #5. Discuss your results. In this exercise, use your own judgment to keep a sufficient number of terms in the infinite series to ensure accuracy of the solution for the plots. Please indicate how you truncate the infinite series.