MAE502 Homework #5

Prob. 1 (6 points)

(a) Using the Fourier transform method, solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad ,$$

for $u(x,t)$ defined on the infinite interval, $x \in (-\infty, \infty)$, and with the boundary condition

$$u(x,0) = \mathrm{P}(x) \; ,$$

where

$$\begin{aligned} \mathrm{P}(x) &= 1 \;\; , \;\; 1 \le x \le 2 \\ &= 1 \;\; , \;\; -2 \le x \le -1 \\ &= 0 \;\; , \;\; \text{otherwise} \; . \end{aligned}$$

(b) Using your solution in (a), evaluate and plot $u(x,t)$ as a function of x at t = 0.1 and 0.3, along with the initial state $u(x,0) = \mathrm{P}(x)$. Discuss the results.
Part (b) is important and accounts for up to 50% of the score.

Note: For (a), a solution in the form of an integral (see Example 1A in slides #15) is acceptable as long as you are able to use it to do the evaluation in (b). For (b), numerical integration may be needed (for example, trapezoidal method should work). Since numerical integration cannot go all the way to ∞, some truncation (replacement of ∞ with a finite bound for the integration) is needed. This is akin to truncating the infinite Fourier series at a finite *n* to do the evaluation and plotting for Prob 2(b) in HW#2. A useful way to determine where to truncate the integral is to plot (for a given *t*) $U(\omega,t)$ as a function of ω and observe how $U(\omega,t)$ decays with ω.

Prob. 2 (4 points)

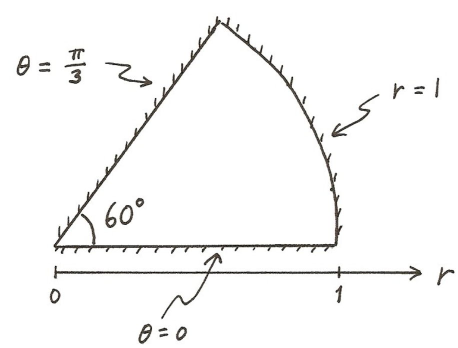
(Modified from Prob. 2.5.7 in textbook)

(a) Solve the Laplace equation,

$$\nabla^2 u = 0 \quad ,$$

for $u(r,\theta)$ defined inside a 60° wedge of radius 1 (i.e., the domain is one-sixth of a full circular disk, see sketch at right) and with the boundary conditions,

(I) $u(r, 0) = 0$
(II) $u(r, \pi/3) = 0$
(III) $u(1,\theta) = \sin(6\theta)$



(b) Plot the solution as a color/contour map.