MAE 502 Homework 6

Prob. 1 (4 points)
Repeat HW2 Prob. 2 but now use numerical method to solve the Laplace equation,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad ,$$

for $u(x, y)$ defined on $x \in [0,1]$, $y \in [0,1]$,with the boundary conditions,

(I) $u(x, 0) = 0$, (II) $u(x, 1) = P(x)$, (III) $u(0, y) = F(y)$, (IV) $u(1, y) = 0$,

where

$P(x) = \sin^2(\pi x)$,

$$F(y) = y \quad , \ 0 \le y \le 1/2$$
$$= 1 - y \ , \ 1/2 < y \le 1 \ .$$

(a) First, use a coarse resolution with $\Delta x = 0.25$, $\Delta y = 0.25$. Compare your solution with the analytic solution from HW2 Prob. 2 for $u(x,t)$ at $(x,t) = (0.25,0.25)$ and $(0.5, 0.75)$.

(b) Repeat (a) but refine the resolution to $\Delta x = 0.1$, $\Delta y = 0.1$. Make a contour/color plot of your solution in the same fashion as HW2 Prob.2. (If the solution is good it should approach the analytic solution as Δx and Δy decrease.)

Prob 2 (4 points)
(a) Using the method of characteristics, solve the nonlinear (or "quasilinear") PDE,

$$\frac{\partial u}{\partial t} + 2u\frac{\partial u}{\partial x} = 0 \quad ,$$

for $u(x,t)$ defined on $x \in (-\infty, \infty)$ and $t \in [0, \infty)$, with boundary condition

$u(x,0) = \mathrm{P}(x)$,

where

$$\mathrm{P}(x) = 1 \quad , \ x < 0$$
$$= 1 + x^2 \quad , 0 \le x \le 1$$
$$= 2 \quad , \ x > 1 \quad .$$

Using your solution, evaluate $u(x,t)$ at $(x = 1, t = 0.05)$ and $(x = 0.1, t = 0.1)$.

(b) Plot the solution, $u(x,t)$, at $t = 0.1$ and $t = 0.2$, along with the "initial state", $u(x,0) = \mathrm{P}(x)$. Briefly discuss the behavior of the solution.