

MAE 502 Homework 6

Prob. 1 (4 points)

Repeat HW2 Prob. 2 but now use numerical method to solve the Laplace equation,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 ,$$

for $u(x, y)$ defined on $x \in [0, 1], y \in [0, 1]$, with the boundary conditions,

$$(I) u(x, 0) = 0, \quad (II) u(x, 1) = P(x), \quad (III) u(0, y) = F(y), \quad (IV) u(1, y) = 0 ,$$

where

$$P(x) = \sin^2(\pi x) ,$$

$$F(y) = y \quad , \quad 0 \leq y \leq 1/2 \\ = 1 - y \quad , \quad 1/2 < y \leq 1 .$$

(a) First, use a coarse resolution with $\Delta x = 0.25, \Delta y = 0.25$. Compare your solution with the analytic solution from HW2 Prob. 2 for $u(x, t)$ at $(x, t) = (0.25, 0.25)$ and $(0.5, 0.75)$.

(b) Repeat (a) but refine the resolution to $\Delta x = 0.1, \Delta y = 0.1$. Make a contour/color plot of your solution in the same fashion as HW2 Prob.2. (If the solution is good it should approach the analytic solution as Δx and Δy decrease.)

Prob 2 (4 points)

(a) Using the method of characteristics, solve the nonlinear (or "quasilinear") PDE,

$$\frac{\partial u}{\partial t} + 2u \frac{\partial u}{\partial x} = 0 ,$$

for $u(x, t)$ defined on $x \in (-\infty, \infty)$ and $t \in [0, \infty)$, with boundary condition

$$u(x, 0) = P(x) ,$$

where

$$P(x) = 1 \quad , \quad x < 0 \\ = 1 + x^2 \quad , \quad 0 \leq x \leq 1 \\ = 2 \quad , \quad x > 1 .$$

Using your solution, evaluate $u(x, t)$ at $(x = 1, t = 0.05)$ and $(x = 0.1, t = 0.1)$.

(b) Plot the solution, $u(x, t)$, at $t = 0.1$ and $t = 0.2$, along with the "initial state", $u(x, 0) = P(x)$. Briefly discuss the behavior of the solution.