Example 1: Find **Fourier Sine** series representation of f(x), defined as

$$f(x) = 1\,, \quad 0 \le x \le 1/2$$
$$\quad = 0\,, \quad 1/2 < x \le 1$$

Step 1: Obtain the odd extension of f(x), for the domain of $-1 \le x \le 1$, as

$$F(x) = 1 \;, \; 0 \le x \le 1/2$$
$$= 0 \;, \; 1/2 < x \le 1$$
$$= -1 \;, \; -1/2 \le x < 0$$
$$= 0 \;, \; -1 \le x < -1/2$$

Step 2: Represent F(x) by the Fourier Sine series for $-1 \le x \le 1$. (This is the case with L = 1 using Eqs. **3.3.1** and **3.3.2** in the textbook)

$$F(x) \approx \sum_{n=1}^{\infty} b_n \sin(n\pi x) \;, \qquad (1)$$

with

$$b_n = \int_{-1}^{1} F(x)\sin(n\pi x)\,dx \;. \qquad (2)$$

Alternativelt, considering both F(x) and sin(nπx) are odd (thus, F(x) sin(nπx) is even), (2) can be rewritten as

$$b_n = 2\int_{0}^{1} F(x)\sin(n\pi x)\,dx \qquad (3)$$

Eq. (3) is Eq. **3.3.6** in textbook. Also see **Sec. 2.3.6** for related discussion.

Using (3), we have

$$b_n = 2\left(\int_0^{1/2} 1 \sin(n\pi x)\,dx + \int_{1/2}^{1} 0 \sin(n\pi x)\,dx \right)$$

$$= 2\int_0^{1/2} \sin(n\pi x)\,dx$$

$$= \frac{2}{n\pi}\left(1 - \cos(\frac{n\pi}{2})\right) \quad .$$

so

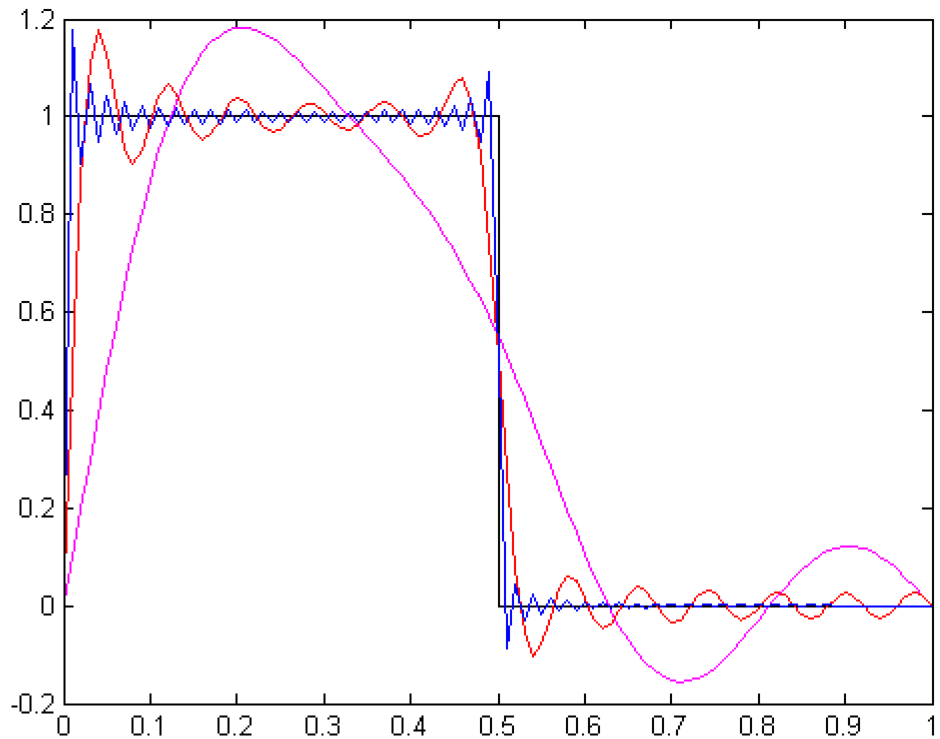
$$b_n = \frac{2}{n\pi} \quad , \text{ if n is odd}$$

$$= \frac{4}{n\pi} \quad , \text{ if mod(n, 4) = 2} \quad \text{(n is even, but not divisible by 4; has remainder of 2)}$$

$$= 0 \; , \quad \text{if mod(n, 4) = 0} \quad \text{(n is divisible by 4)}$$

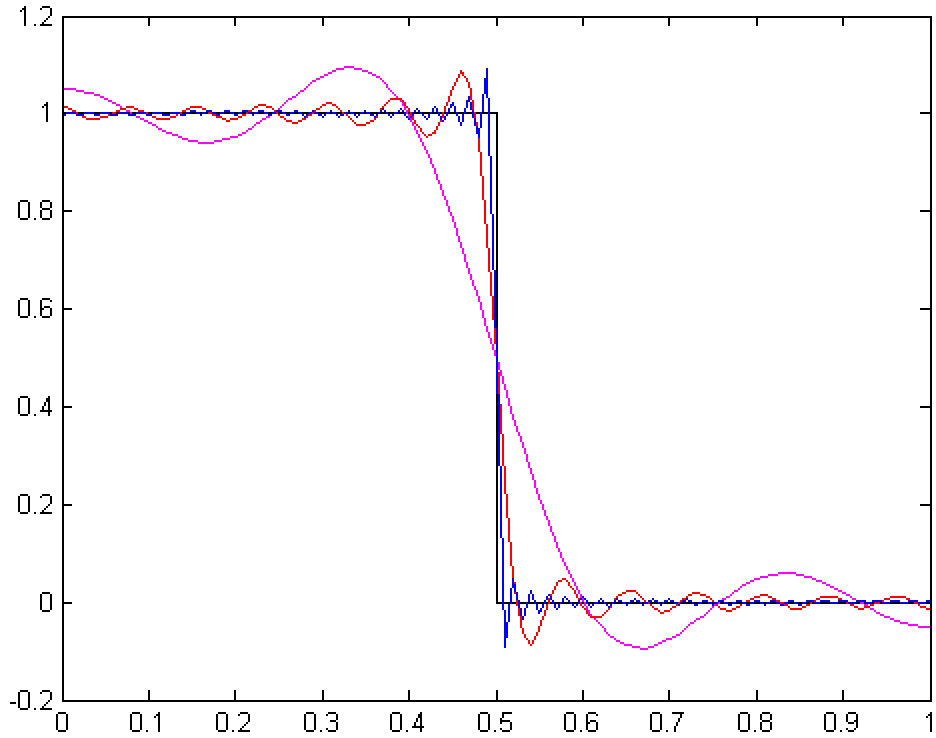
Note: mod(p,q) is the remainder of p divided by q. For example, mod(10,3) = 1, because 10 = 3 x 3 +1. This is an extremely useful function especially for Matlab programming. See Matlab Example #4.

The plot in the next page shows the result of Fourier Sine series expansion with the series truncated at n = 5, 25, and 100. The relevant Matlab code can be found in Matlab Example #4.



Fourier Sine series representation

Example 2: Find **Fourier Cosine** series representation of the same f(x) in Example 1. This is left to you as an exercise. The results with the series truncated at n = 5, 25, and 100 are shown below.



Fourier Cosine series representation