Example 1: Find **Fourier Sine** series representation of f(x), defined as

$$
\begin{aligned}
f(x) &= 1\,, \quad 0 \le x \le 1/2 \\
&= 0\,, \quad 1/2 < x \le 1
\end{aligned}
$$

Step 1: Obtain the odd extension of f(x), for the domain of $-1 \le x \le 1$, as

$$
\begin{aligned}
F(x) &= 1 \quad, \; 0 \le x \le 1/2 \\
&= 0 \quad, \; 1/2 < x \le 1 \\
&= -1 \;, \; -1/2 \le x < 0 \\
&= 0 \quad, \; -1 \le x < -1/2
\end{aligned}
$$

Step 2: Represent F(x) by the Fourier Sine series for $-1 \le x \le 1$. (This is the case with L = 1 using Eqs. **3.3.1** and **3.3.2** in the textbook)

$$F(x) \approx \sum_{n=1}^{\infty} b_n \sin(n\pi x) \;, \qquad (1)$$

with

$$b_n = \int_{-1}^{1} F(x) \sin(n\pi x)\, dx \;. \qquad (2)$$

Alternativelt, considering both F(x) and sin(nπx) are odd (thus, F(x) sin(nπx) is even), (2) can be rewritten as

$$b_n = 2\int_{0}^{1} F(x) \sin(n\pi x)\, dx \qquad (3)$$

Eq. (3) is Eq. **3.3.6** in textbook. Also see **Sec. 2.3.6** for related discussion.

Using (3), we have

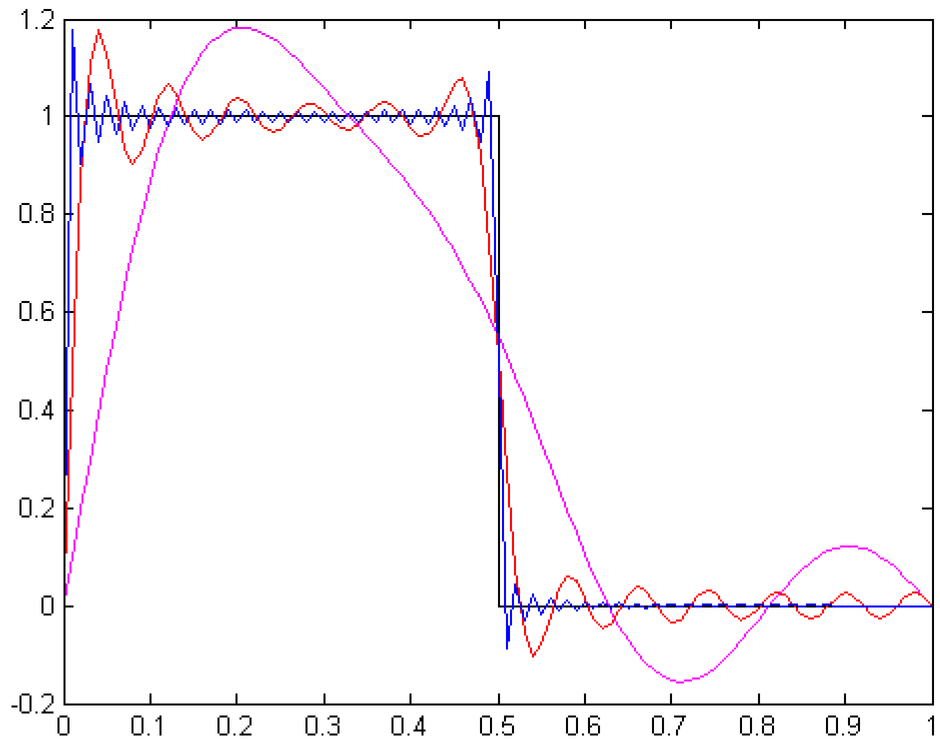
$$
\begin{aligned}
b_n &= 2\left(\int_0^{1/2} 1\sin(n\pi x)\,dx + \int_{1/2}^{1} 0\sin(n\pi x)\,dx\right) \\
&= 2\int_0^{1/2} \sin(n\pi x)\,dx \\
&= \frac{2}{n\pi}\left(1-\cos(\frac{n\pi}{2})\right) \quad .
\end{aligned}
$$

so

$$
\begin{aligned}
b_n &= \frac{2}{n\pi} \;, \text{ if } n \text{ is odd} \\
&= \frac{4}{n\pi} \;, \text{ if } \mathrm{mod}(n, 4) = 2 \quad \text{(n is even, but not divisible by 4; has remainder of 2)} \\
&= 0 \;, \quad \text{ if } \mathrm{mod}(n, 4) = 0 \quad \text{(n is divisible by 4)}
\end{aligned}
$$

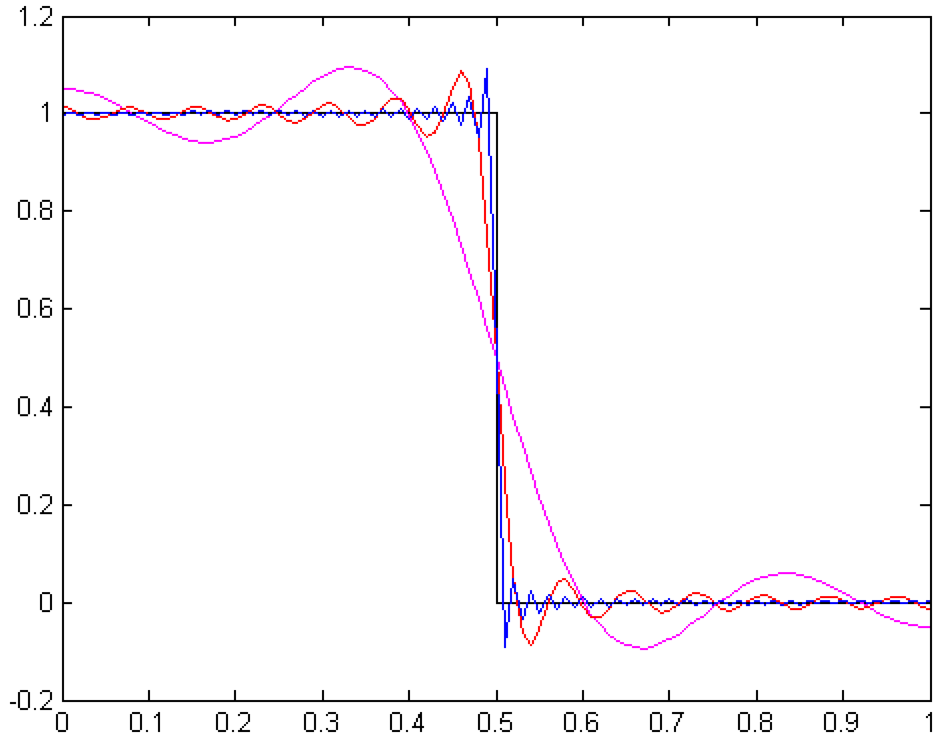
Note: mod(p,q) is the remainder of p divided by q. For example, mod(10,3) = 1, because 10 = 3 x 3 +1. This is an extremely useful function especially for Matlab programming. See Matlab Example #3.

The plot in the next page shows the result of Fourier Sine series expansion with the series truncated at n = 5, 25, and 100. The relevant Matlab code can be found in Matlab Example #3.



Fourier Sine series representation

Example 2: Find **Fourier Cosine** series representation of the same f(x) in Example 1. This is left to you as an exercise. The results with the series truncated at n = 5, 25, and 100 are shown below.



Fourier Cosine series representation