Lecture 8

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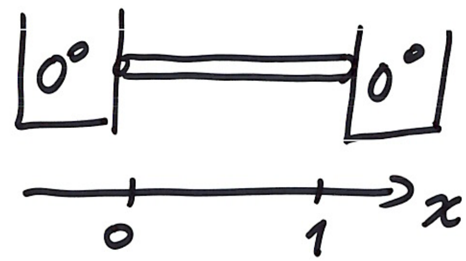
* Tips on coding for HW1-Q1

"First Example" of end-to-end sol. (Lec 4, 5)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \qquad 0 \le x \le 1, \quad t \ge 0$$



$u(0,t) = 0$

$u(1,t) = 0$

$u(x,0) = P(x) \leftarrow$ say $P(x) = [\sin(\pi x)]^8$

say, we desire to obtain solution at $t = 0.01$ and plot the sol. as a function of x.

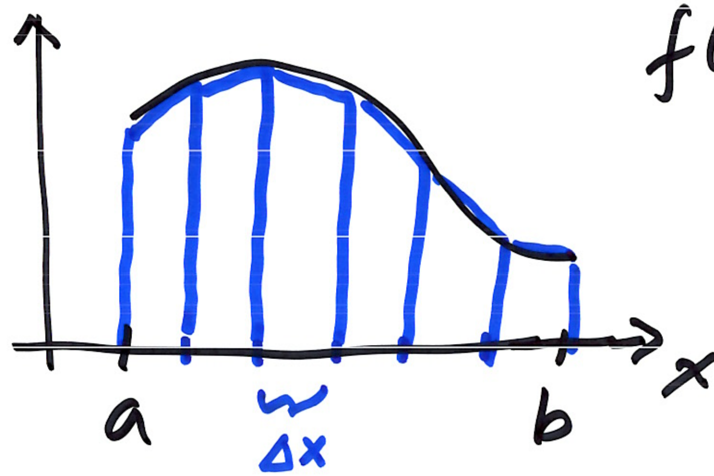
Full sol.

$$\begin{cases} u(x,t) = \displaystyle\sum_{n=1}^{\infty \to N} a_n \sin(n\pi x)\, e^{-(n\pi)^2 t} \\ a_n = 2\displaystyle\int_0^1 P(x)\sin(n\pi x)\,dx \qquad n = 1, 2, 3, \dots \end{cases}$$

Matlab code


```
dx = 0.01;
x = [0:dx:1];
p = sin(pi*x).^8;
N = 10;
for n=1:N
    a(n) = 2*trapz(x, p.*sin(n*pi*x));
end
t = 0.01;
u = 0*x;
for n=1:N
    u = u + a(n)*sin(n*pi*x)*exp(-((n*pi)^2)*t);
end
plot(x, u)
```

trapezoidal method ("trapz" in matlab)

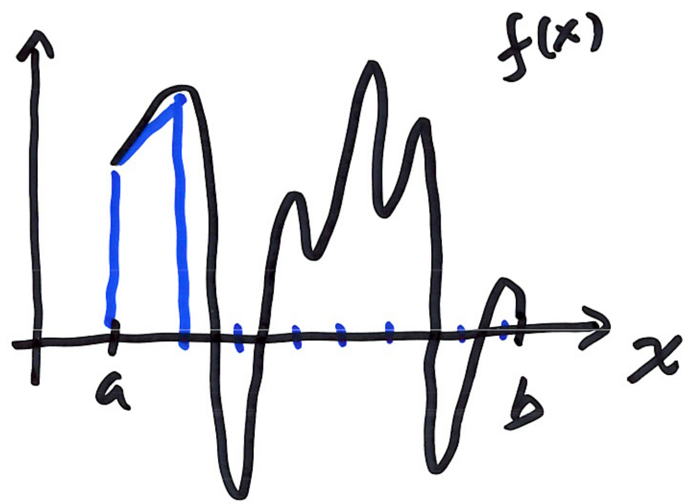


V

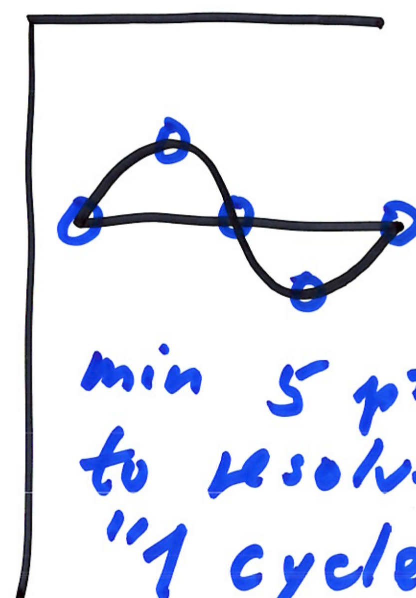
$f(x)$

$$I = \int_a^b f(x)\,dx$$

=



X



min 5 pt to resolve "1 cycle"

10 pts recom.

$$\sum_{n=1}^{\infty \; N} a_n \sin(\underline{n}\pi x) \;\ldots$$

$\sin(1\pi x)$

$x=0$ $x=1$ $n=1$

$\sin(5\pi x)$

$n=5$

$N=100$

0 ... 1

$\sin(100\pi x)$

Need $100 * 10$

~ 1000

grid pt

$$u(x,t) = \sum \underbrace{a_n \underbrace{\sin(n\pi x)}_{\text{bounded by } \pm 1} \underbrace{e^{-(n\pi)^2 t}}_{\text{bounded by } 1}}$$

$$|e^{-x}| \leq 1$$

Other tips

HW1 <u>Q2</u>, <u>Q3</u> : We require <u>exact</u> sol. with only a finite # of terms.

Revisit "First example"

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

(I) — $u(0,t)=0$

(II) — $u(1,t)=0$

(III) — $\underline{u(x,0)=P(x)}$ say, $\underline{P(x) = \sin(\pi x) + 5\sin(3\pi x)}$

Full sol. $u(x,t) = \sum_{n=1}^{\infty} a_n \sin(n\pi x)\, e^{-(n\pi)^2 t}$ — (*)

$a_n = 2\int_0^1 P(x)\sin(n\pi x)\,dx \qquad n=1,2,3,\ldots$

at $\underline{t=0 \quad (*)}$ + $\underline{\text{b.c. (III)}}$

$$\sum_{n=1}^{\infty} a_n \sin(n\pi x) = p(x) = \sin(\pi x) + 5\sin(3\pi x)$$

$$\underline{a_n = ?}$$

$$a_1 \sin(\pi x) + a_2 \sin(2\pi x) + a_3 \sin(3\pi x) + a_4 \sin(4\pi x) + \ldots$$

$$= \sin(\pi x) + 5 \sin(3\pi x)$$

"visual inspection"

$\underline{a_1 = 1, \quad a_3 = 5,}$ all other $\underline{a_n = 0}$

Exact Full sol: $(*)$ with

$$u(x,t) = \sin(\pi x) e^{-\pi^2 t} + 5 \sin(3\pi x) e^{-9\pi^2 t}$$

#

HW1 Q4 : has nonhomogeneous b.c.'s

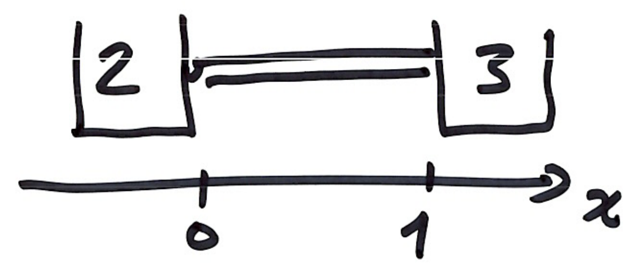
BUT we only ask for

<u>steady solution</u>

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

$$u(0,t) = 2$$
$$u(1,t) = 3$$
$$u(x,0) = p(x)$$

$u_s(x)$



$$\frac{d^2 u_s}{dx^2} = 0 \qquad u_s(0) = 2 \qquad u_s(1) = 3$$