# Lecture 5

9/17

**HW1**: Work on Tasks 1-3 (NOT 1-4) ← typo

↳ Task 1, Task 2 } Already covered in Lecture 3, 4

* Use in-class demos to override inaccuracies in Tutorial

Task 3 — ~~Today~~ Lec 6

HW1 (Tutorial 1, 2)

* Incompressible flow w/ some heat transfer or thermal processes

↳ "pressure based solver" in Fluent

Fluent: ☑ Energy eq.

* $\rho$ = const. (This is artificial!)

Recap: Lecture 2 (governing eqs.)

Newton's 2nd law $\leftarrow$ Momentum eq. $u$, $v$, $w$

$$\frac{d\vec{v}}{dt} = \frac{\vec{F}}{m}$$

(Mass) Continuity eq. $\rho$ (*)

$$\frac{dm}{dt}$$

(Thermal) Energy eq. ?

1st law of thermodyn.

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Lecture 2 :

Lagrangian

Eulerian ✓ CFD

"T"

fields of $u, v, w, \rho, \ldots$

$$\frac{d()}{dt} \equiv \frac{\partial()}{\partial t} + u\frac{\partial()}{\partial x} + v\frac{\partial()}{\partial y} + w\frac{\partial()}{\partial z}$$

Ex: (Lec 2)

$$\cancel{\partial} \quad \frac{dT}{dt} \equiv \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} \quad \checkmark$$

(Diffusive processes are excluded) ⟶ * Viscous effect for momentum

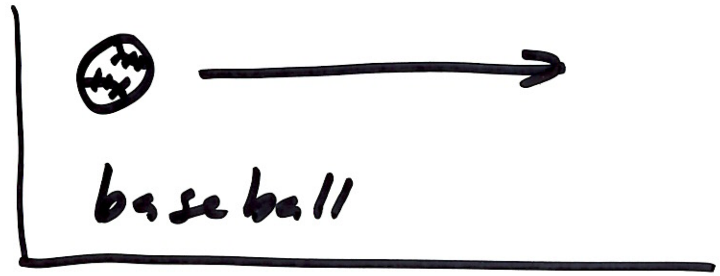
* Molecular heat conduction

add visc. terms to momenta

add one term to energy

Fluid

$\mathcal{M}$ of parcel




$$\mathcal{M} \equiv \rho \cdot V$$

attempt to write out the Eulerian eq. for $\rho$

$$\frac{d\mathcal{M}}{dt} = 0 \Rightarrow \frac{d}{dt}(\rho V) = 0 \qquad \rho \frac{dV}{dt} + V \frac{d\rho}{dt} = 0$$

$$\frac{1}{V}\frac{dV}{dt} = -\frac{1}{\rho}\frac{d\rho}{dt}$$

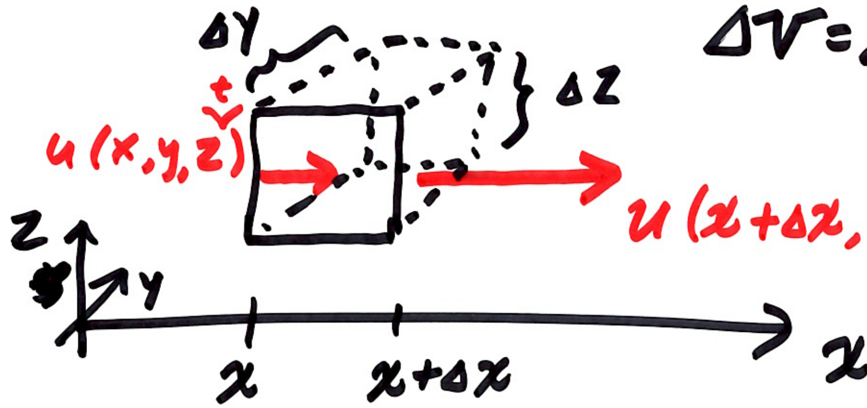
$$\frac{\Delta V}{V} \cdot \frac{1}{\Delta t} \approx -\frac{\Delta \rho}{\rho} \cdot \frac{1}{\Delta t}$$

more kinematics

1-D

$$\Delta V = \Delta x \cdot \Delta y \cdot \Delta z$$

$t=0$




wait for $\Delta t$

$t=\Delta t$

Amount of stretching in x-dir:


$\delta$

$$\underline{\delta x} = [u(x+\Delta x, y, z, t) - u(x,y,z,t)]\,\Delta t$$

$\underline{\text{change of volume}}$

$$= \underline{\delta x} \cdot \Delta y \cdot \Delta z$$

$$\frac{\Delta V}{V} = \frac{\delta x \cdot \cancel{\Delta y} \cdot \cancel{\Delta z}}{\Delta x \cdot \cancel{\Delta y} \cdot \cancel{\Delta z}}$$

$$\frac{\Delta V}{V} = \frac{[u(x+\Delta x, y, z, t) - u(x, y, z, t)] \cdot \Delta t}{\Delta x}$$

$$\Rightarrow \quad \frac{1}{\Delta t}\frac{dV}{V} = \frac{u(x+\Delta x, y, z, t) - u(x, y, z, t)}{\Delta x}$$

$$\frac{1}{V}\frac{\Delta V}{\Delta t}$$

$$\lim_{\substack{\Delta x \to 0 \\ \Delta t \to 0}} \qquad \frac{1}{V}\frac{dV}{dt} = \frac{\partial u}{\partial x}$$

Repeat in y-dir $\quad \frac{1}{V}\frac{dV}{dt} = \frac{\partial v}{\partial y}$

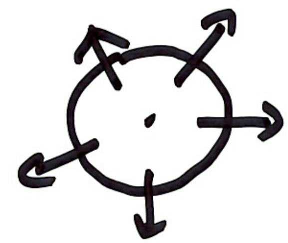
Repeat in z-dir: $\quad \frac{1}{V}\frac{dV}{dt} = \frac{\partial w}{\partial z}$

Full 3-D kinematics

$$\frac{1}{V}\frac{dV}{dt} = \underbrace{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}}_{\nabla \cdot \vec{V}} \quad *$$

$\nabla \cdot \vec{V} < 0$

"convergence"



$\nabla \cdot \vec{V} > 0$

"divergence"

Recall $\frac{d\mathcal{M}}{dt} \longrightarrow \frac{1}{V}\frac{dV}{dt} = -\frac{1}{\rho}\frac{d\rho}{dt}$

replace it by (*)

$$\Rightarrow \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{d\rho}{dt} \quad \text{--- (1)}$$

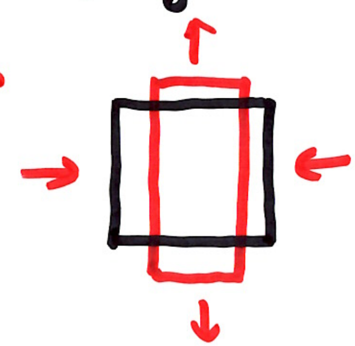
"Incompressible" by definition : $\frac{d\rho}{dt} = 0$

$\left(\frac{d\rho}{dp} = 0\right)$

then

Lagrangian

"d", NOT "$\partial$"



* Incompressible flow $\frac{d\rho}{dt} = 0$

$\Rightarrow$ (1) becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{— (2)}$$

Full eq. of conserv. of mass

"$\rho$" disappears

And it only involves velocity !! !

This is used in Hw1 & Tutorial 1,2

No need to compute density

$\rho$-based $\longrightarrow$ $p$-based solver

In general (Compressible)

$$\frac{d\rho}{dt} \equiv \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z}$$

↳ plug it into (1) :

Full continuity eq :

$$\frac{\partial \rho}{\partial t} = -\left(u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z}\right) - \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

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$\rho$ - based solver in Fluent

**Energy eq:** 1st law of thermo.

$$đq = c_p\, dT + \cancel{p\, dV} \quad -V\,dp$$

correction (this doesn't affect the rest of discussion)

Incompressible flow $\underline{dV = 0}$

Adiabatic $đq = 0$

$$\frac{dT}{dt} = 0 \quad \Longrightarrow \quad \underbrace{\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} - w\frac{\partial T}{\partial z}}$$

HW 1 (Tutorial 1)

$\rho = 0$

~~con~~ <u>HW1</u>

mass $$0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \quad \text{——} \quad (**)$$

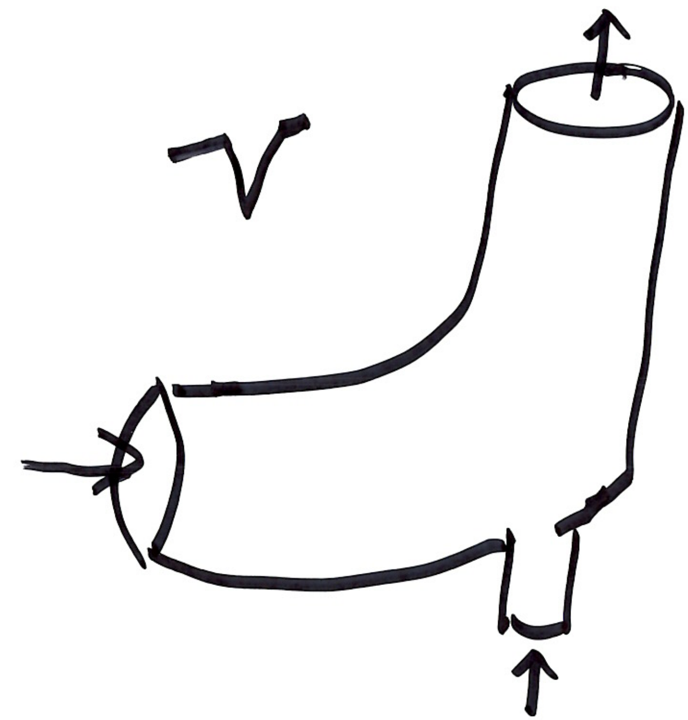
$$\left( 0 = \nabla \cdot \vec{V} \right)$$

energy $$\frac{\partial T}{\partial t} = \underbrace{- u \frac{\partial T}{\partial x} - v \frac{\partial T}{\partial y} - w \frac{\partial T}{\partial z}}_{- \vec{V} \cdot \nabla T}$$

$$\frac{\partial (uT)}{\partial x} = u \frac{\partial T}{\partial x} + T \frac{\partial u}{\partial x}$$

$$= \underbrace{- \frac{\partial (uT)}{\partial x} - \frac{\partial (vT)}{\partial y} - \frac{\partial (wT)}{\partial z}}_{- \nabla \cdot (\vec{V} T)}$$

$$\frac{\partial T}{\partial t} = -\underbrace{\nabla \cdot (\vec{V} T)}$$




Total (thermal) energy → this is enthalpy

$$E \equiv \iiint_{V} c_p \cdot \rho \, T \, dV$$

HW1 Task 3

Cons. of energy

$$\frac{dE}{dt} \equiv \frac{d}{dt} \iiint_{V} c_p \rho T \, dV$$

$$\equiv \iiint_{V} c_p \rho \frac{\partial T}{\partial t} \, dV$$

$$= - c_p \cdot \rho \cdot \iiint_{V} \nabla \cdot (\vec{V} T) \, dV$$

$$= - c_p \cdot \rho \oiint_{S} (\vec{V} T) \cdot \hat{n} \, dS$$