Lecture 6

9/9

Today: "Energy budget" in HW1

$\hookrightarrow$ <u>Task 3</u>

Recap (Lec 5) <u>Mass eq.</u> (continuity eq.)

Fully <u>compressible flow</u>:

An eq. of <u>density</u>

$$\frac{\partial \rho}{\partial t} = -\left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}\right) - \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

Fluent $\rightarrow$ "Density based solver"

<u>Incompressible flow</u> $\underline{\left(\frac{d\rho}{dt} = 0\right)}$, (Special case: $\rho = \text{const}$, HW1)

ρ vanishes entirely

$$\underline{0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}} \rightsquigarrow \nabla \cdot \vec{V} = 0 \quad (1)$$

Fluent $\rightarrow$ "Pressure based solver"

☑ Energy eq. (1st law of thermo.)

$$\cancel{\delta q} \xrightarrow{} 0 \quad = c_p\, dT \; \cancel{+ p\,dV} \xrightarrow{} \text{incompressible}$$

Correction: should be "$-V dp$" (this doesn't affect the rest of discussion)

<u>Hw 1</u> (No heat source/sink)

$\rho \equiv \text{const}$

$$\boxed{\frac{dT}{dt} = 0} \quad \text{—— (2)}$$

(L) → (E)

$$\cancel{\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}}$$

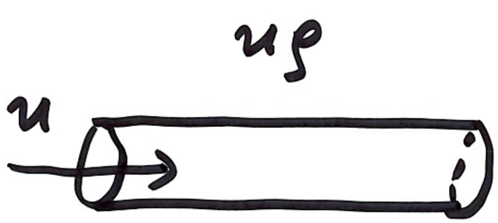
$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = 0 \quad \text{—— (2*)}$$

2* : $$\frac{\partial T}{\partial t} = -\underline{\vec{V}\cdot\nabla T}$$

1 : $$\underline{\nabla\cdot\vec{V} = 0}$$

$$\underbrace{u\frac{\partial T}{\partial x}}_{\frac{\partial (uT)}{\partial x} - T\frac{\partial u}{\partial x}} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \overbrace{\frac{\partial (uT)}{\partial x} + \frac{\partial (vT)}{\partial y} + \frac{\partial (wT)}{\partial z}}^{\nabla\cdot(\vec{V}T)} \cancel{- T\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)} \xrightarrow{\text{by } 1} 0$$

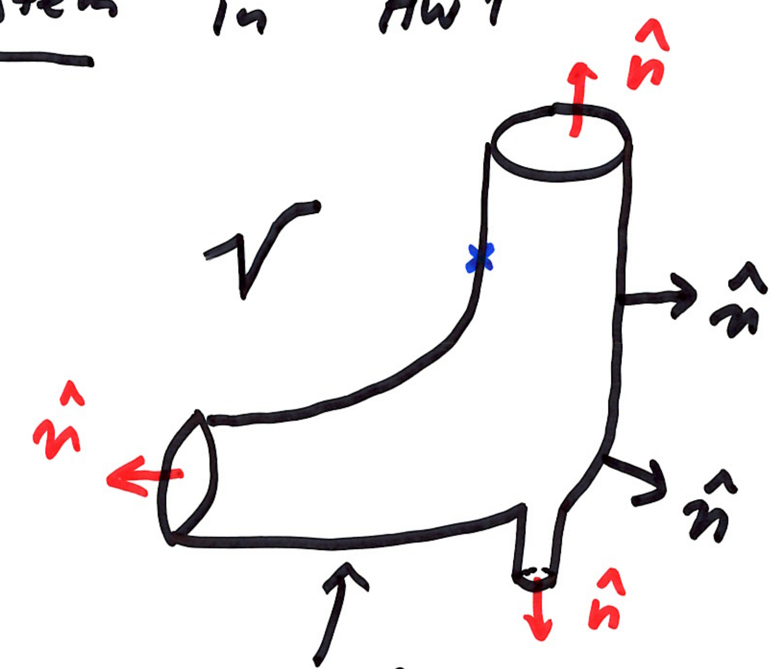
$$\frac{\partial T}{\partial t} = -\nabla\cdot(\underbrace{\vec{V}T}_{\text{"energy flux"}}) \qquad \text{--- } 3$$



Total thermal / internal Energy for the whole system in Hw 1

correction: should be "enthalpy"

$$E \equiv \iiint_{\mathcal{V}} c_p \cdot \rho \cdot T \, d\mathcal{V}$$



sfc of $\mathcal{V}$ is S

$$\cancel{\frac{dE}{dt}} \equiv \frac{d}{dt} \iiint_{\mathcal{V}} c_p \rho T \, d\mathcal{V}$$

Steady sol.

$$= c_p \cdot \rho \cdot \frac{d}{dt} \iiint_{\mathcal{V}} T \, d\mathcal{V}$$

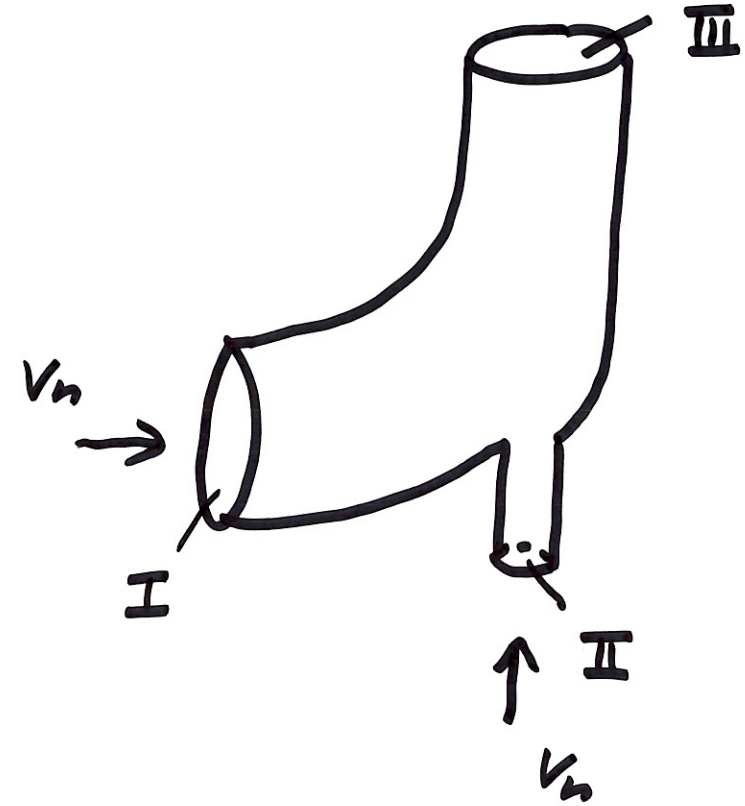
Calculus

$$= c_p \cdot \rho \iiint_{\mathcal{V}} \frac{\partial T}{\partial t} \, d\mathcal{V}$$

Energy eq. $\rightarrow$ $$= - c_p \cdot \rho \iiint_{\mathcal{V}} \nabla \cdot (\vec{V} T) \, d\mathcal{V}$$

Gauss div. $\rightarrow$ $$= - c_p \cdot \rho \oiint_{S} (\vec{V} T) \cdot \hat{n} \, dS$$

$$c_p \cdot \rho \oiint_S (\vec{V} T) \cdot \hat{n} \, dS = 0$$



$$\mathcal{J}_I \equiv \iint_I c_p \cdot \rho \cdot \underline{\underline{V_n}} \, T \, ds$$

$$\mathcal{J}_{II} \equiv \iint_{II} c_p \cdot \rho \cdot \underline{\underline{V_n}} \cdot T dS$$

$$\mathcal{J}_{III} \equiv \iiint_{III} c_p \cdot \rho \cdot \underline{\underline{V_n}} \cdot T dS$$

$$V_n \equiv \vec{V} \cdot \hat{n}$$

We anticipate: $\underline{\underline{\mathcal{J}_I}} + \underline{\underline{\mathcal{J}_{II}}} \approx \underline{\underline{\mathcal{J}_{III}}}$

small discrepancy due to num. error

HW 1 - Task 3

Another source of discrepancy : $g_{I} + g_{II} \approx g_{III}$

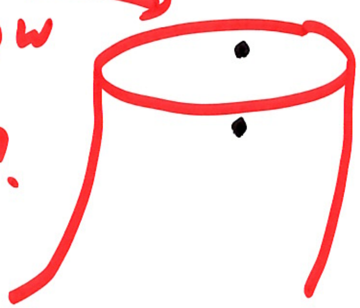
we ignore molecular heat conduction
(which is INCLUDED in Fluent)
!

$$\frac{\partial T}{\partial t} = - \nabla \cdot (\vec{V} T) + K \nabla^2 T$$

$$\cancel{\frac{dE}{dt}} = \underbrace{\iiint_V - \nabla \cdot (\vec{V} T) \, dV}_{g_I,\, g_{II},\, g_{III}} + \underbrace{\boxed{\text{extra}}}_{g_{IV}}$$

For HW1, it's very small

Impose backflow T 290K ?

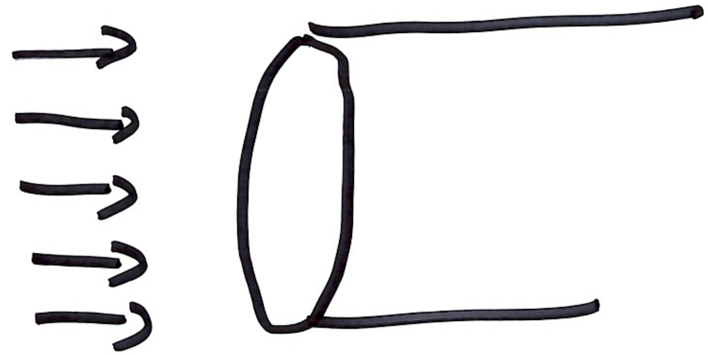


III outlet

Computing $\mathcal{J}_{I}$, $\mathcal{J}_{II}$: simple: u_{inlet} const

T_{inlet} const

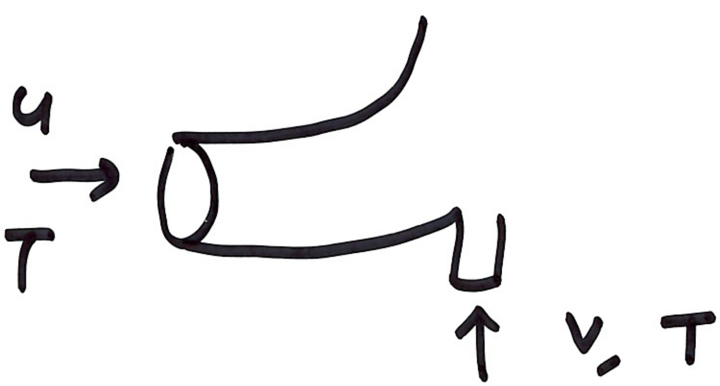
$u = 0.4 \frac{m}{s}$



I

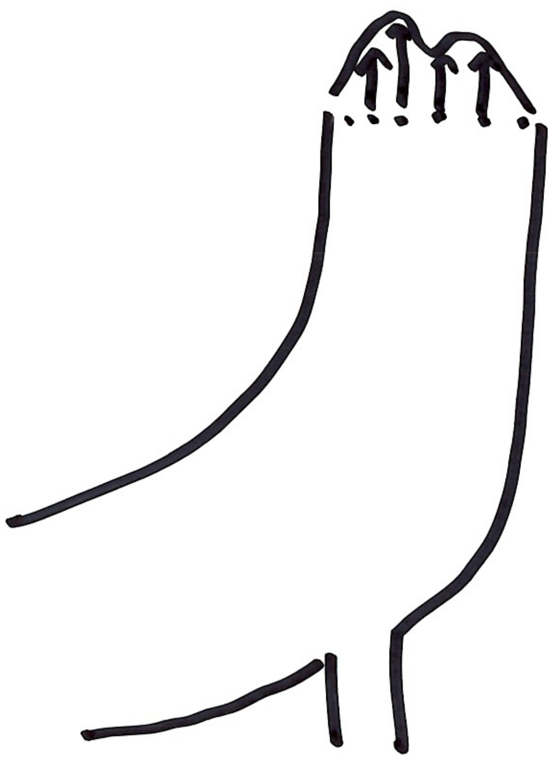
$T = ??\ ^\circ K$

$$\iint_I u T \, dS = u \cdot T \cdot (\text{area})$$



Fluent

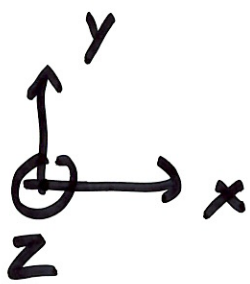
Mesh w/ finite resolution



$u(x, z)$

$T(x, z)$

$$\iint_{III} u(x\ldots)\,T(x\ldots)\,dS$$



$$\frac{\iint T\,dS}{}\ \ II$$

Fluent:

Step 1: Define

$F \equiv u \cdot T$

~~Gen~~ Custom Field Function (Tutorial #2)

Step 2: F

Fluent: Surface Integral

* Convergence criterion — Residual
(Scaled)

Steady sol.

Fluent : Iterations

eqs.: $\underbrace{\square + \square + \square}$ = $\underbrace{\square + \square}$

Ex: $x^2 + 2x = 3$

1st iteration $x = 2$

$2^2 + 2 \cdot 2 = 3$

LHS 8

$|8 - 3| = \underline{\underline{5}}$

residual

2nd iter $x = 1.5$

$(1.5)^2 + 2 \cdot (1.5) = 3$

$2.25 + 3$

5.25

2.25