Lecture 7

9/14

Reminder: **Hw1** due (at Canvas) 11:59 PM, Friday.

office hrs: Mon/Tue 3-5 PM, ERC 359

Figures should be clearly labeled.

Background (governing eqs.)

Relevant points concerning Fluent

- p-based vs. ρ-based solver
- Turbulence vs. Laminar model
- "Scaled residual"
- Steady vs. transient ~~run~~ solution

* Continuity eq. (Cons. of <u>mass</u>) ✓

* <u>Energy eq.</u> (" of thermal <u>energy</u>) 1/2
(static)

discussed special case w/ $\rho = \text{const}$

* Momentum eq. (" of <u>momentum</u>)

Newton's 2nd law

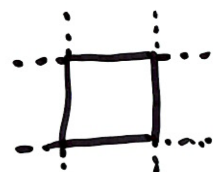
 $\vec{V}$

m

mech. 101

$$\frac{d\vec{V}}{dt} = \frac{\vec{F}}{m}$$

exchange of momentum

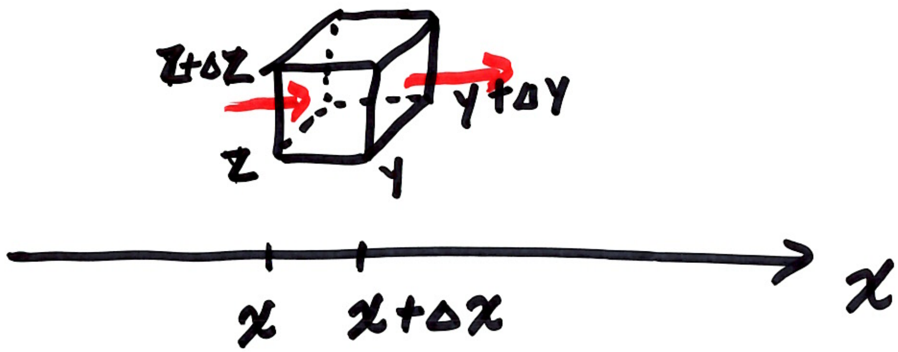


fluid

$P = \frac{F}{A}$

* internal <u>interaction</u> between neighboring parcels, in the "normal" direction

* <u>external force</u> (e.g. gravity)



$\vec{V} = (\underline{\underline{u}}, v, w)$

momentum balance — Net force

force at x : $p(\underline{\underline{x}}, y, z, t) \cdot \Delta y \cdot \Delta z$ (left)

$-)$ " $x+\Delta x$: $p(\underline{x+\Delta x}, y, z, t) \cdot \Delta y \cdot \Delta z$ (right)

$[p(x,y,z,t) - p(x+\Delta x, y, z, t)] \cdot \cancel{\Delta y} \cdot \cancel{\Delta z}$ net force in x-dir

$= \underbrace{\rho \cdot \Delta x \cdot \cancel{\Delta y} \cdot \cancel{\Delta z}}_{\text{mass}} \cdot \underbrace{\left(\frac{du}{dt}\right)}_{\text{acc. in x-dir}}$

$\frac{du}{dt}$

$$\frac{du}{dt}\cancel{\frac{\partial u}{\partial t}} = -\frac{1}{\rho}\left(\frac{p(x+\Delta x, y, z, t) - p(x, y, z, t)}{\Delta x}\right)$$

$$\lim_{\Delta x \to 0} \quad \frac{du}{dt}\cancel{\frac{\partial u}{\partial t}} = \underbrace{-\frac{1}{\rho}\overbrace{\frac{\partial p}{\partial x}}^{\text{pressure gradient}}}_{\text{pressure gradient force}} \quad \text{—— (1)}$$

PGF

Repeat for y- z-direction:

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} \quad \text{—— (2)}$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} \quad \text{—— (3)}$$

Next L $\rightsquigarrow$ E $\qquad \dfrac{d(\,)}{dt} \equiv \dfrac{\partial(\,)}{\partial t} + \underbrace{u\dfrac{\partial(\,)}{\partial x} + v\dfrac{\partial(\,)}{\partial y} + w\dfrac{\partial(\,)}{\partial z}}$

1 $\rightarrow$

$$\left\{\begin{aligned}
\frac{\partial u}{\partial t} &= -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z} - \frac{1}{\rho}\frac{\partial p}{\partial x}\\
\frac{\partial v}{\partial t} &= -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - w\frac{\partial v}{\partial z} - \frac{1}{\rho}\frac{\partial p}{\partial y}\\
\frac{\partial w}{\partial t} &= -\underbrace{u\frac{\partial w}{\partial x} - v\frac{\partial w}{\partial y} - w\frac{\partial w}{\partial z}}_{\text{inertial terms}} - \underbrace{\frac{1}{\rho}\frac{\partial p}{\partial z}}_{\text{P.G.F.}} - g
\end{aligned}\right.$$

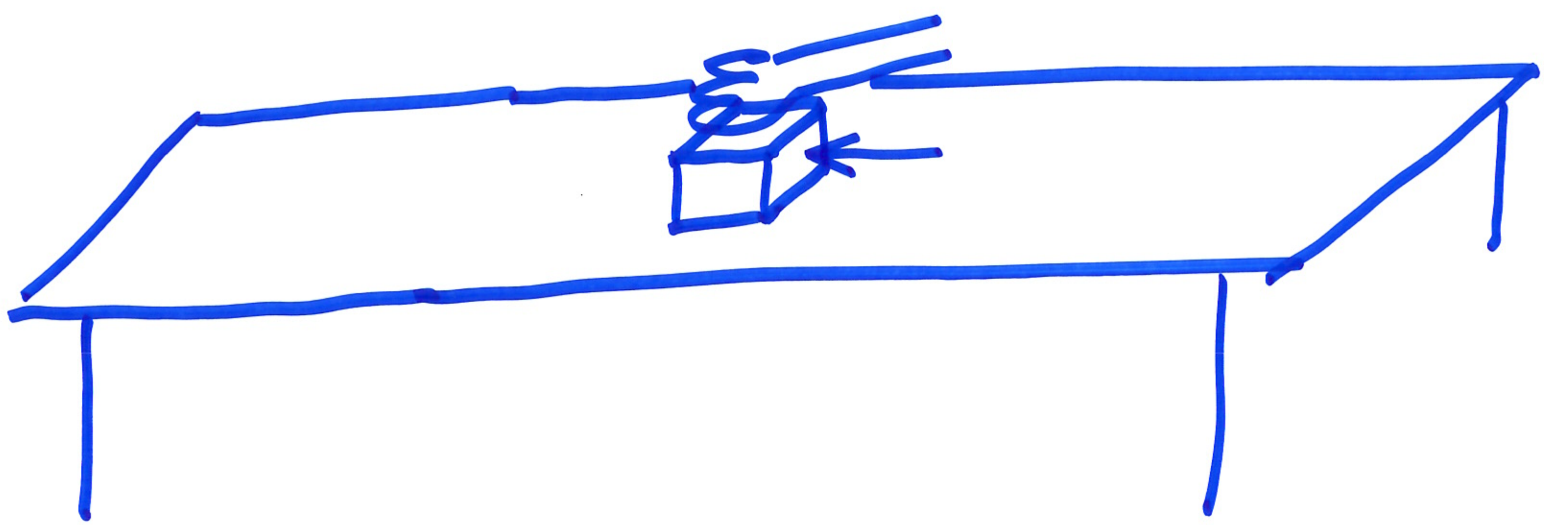
gravity

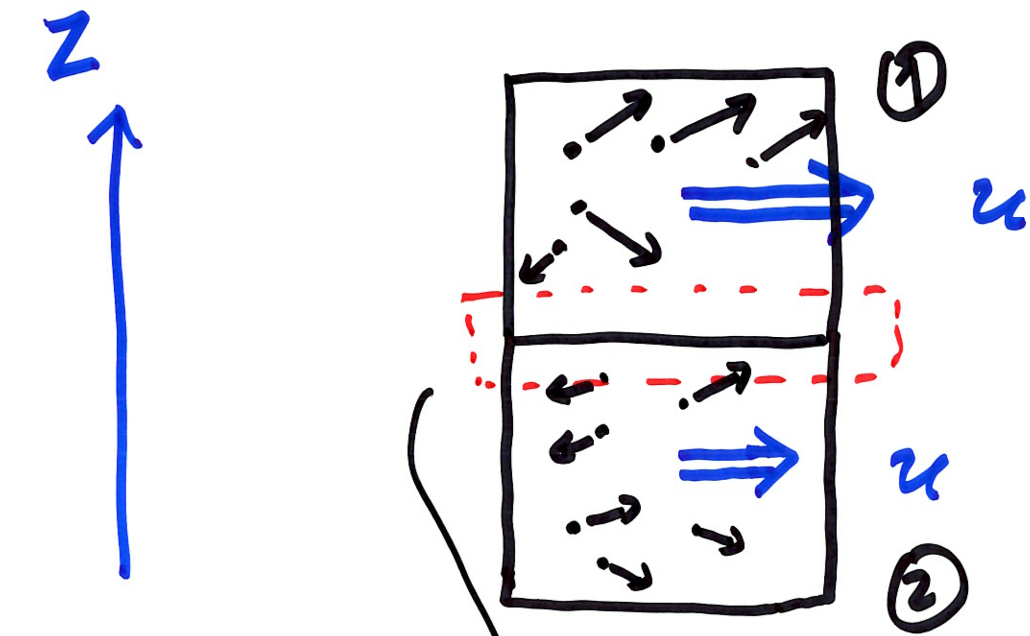
Euler eq.

Momentum eq. for an inviscid flow

Viscous term: "Diffusion" of momentum

internal interaction between neighboring parcels, in the tangential direction

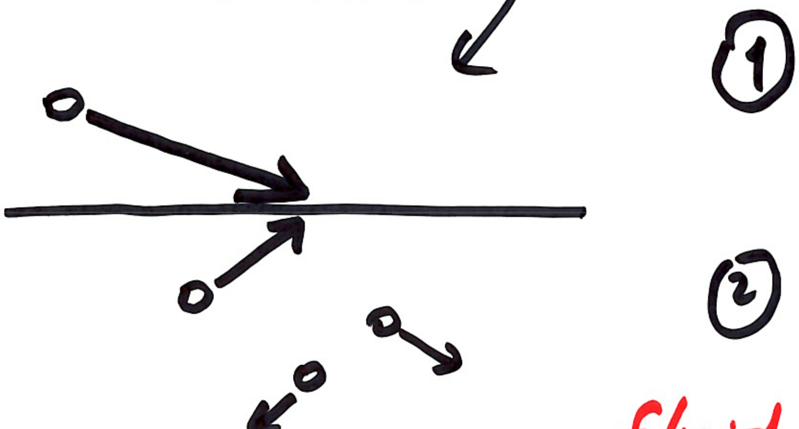


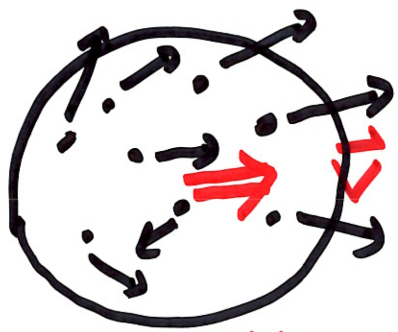


Consider
u vel.

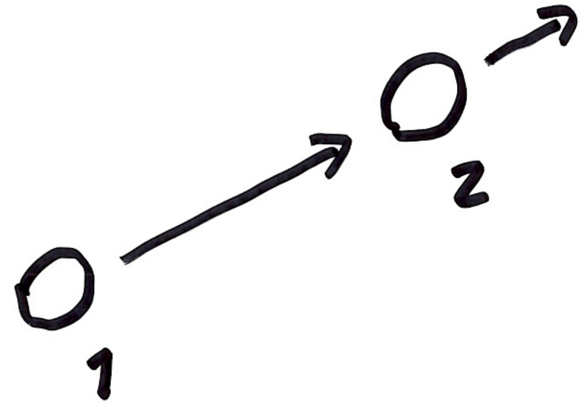
fluid
"parcel"
=
trillions
of
molecules

~~vect~~
"velocity" of
parcel
=
ave. of
molecular
velocity

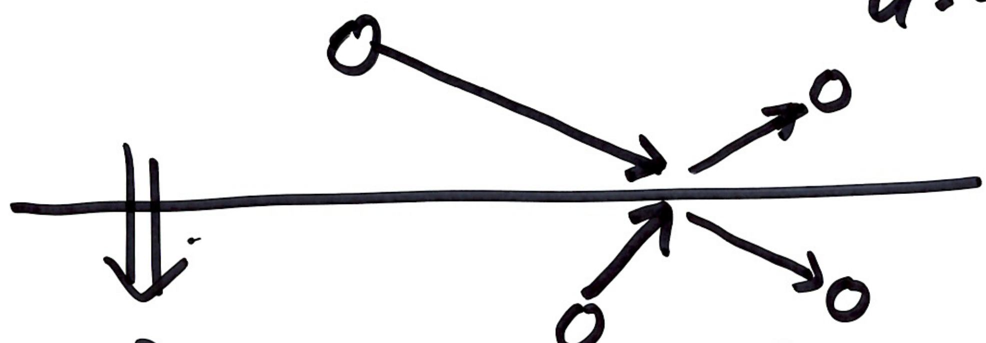




fluid $\vec{V} = \frac{1}{N}\sum_{i=1}^{N} \vec{V}_{m_i}$



(Newtonian fluid)



loss of u-m ①

gain of u-moment ②

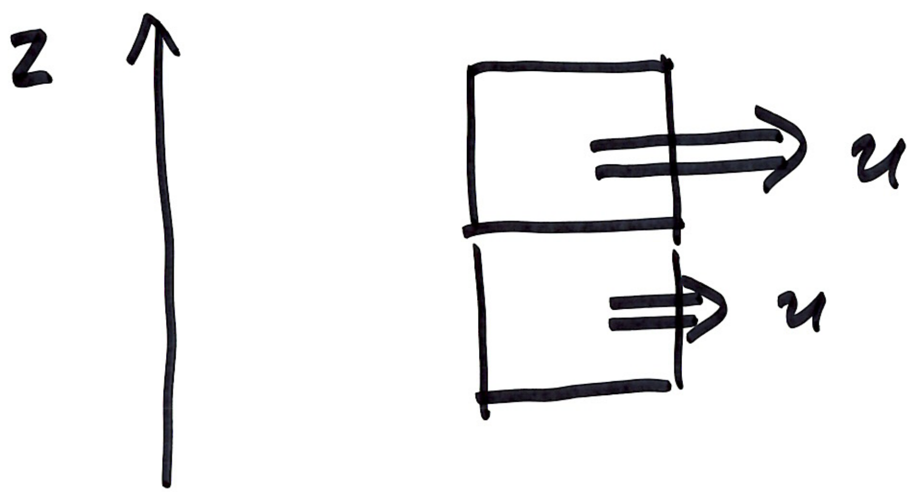
$$\Downarrow$$

$$\phi \propto -\frac{\partial u}{\partial z}$$

$$\phi = -\mu \frac{\partial u}{\partial z}$$

(dynamic) viscosity

Fluent Material



$\Downarrow$ net transfer of u momentum

ϕ : momentum flux

$\left[\phi > 0 \text{ ~~in~~ if } \Rightarrow \text{ is in } +z \text{ dir.} \right]$

$$\frac{\partial u}{\partial t} = \frac{-1}{\rho} \frac{\partial \phi}{\partial z}$$

momentum = $\phi \cdot (\text{time}) \cdot (\text{area})$

amount of momentum passing through x-sec per unit time " " area

$$\frac{\partial u}{\partial t} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial z^2}$$

$\leftarrow$ x-dir

$\leftarrow$ z-dir

Repeat it 8 more times

Navier – Stokes eq (viscous flow) 9 terms

$$\frac{\partial u}{\partial t} = \underbrace{-u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z}}_{\text{Inertial}} \underbrace{- \frac{1}{\rho}\frac{\partial p}{\partial x}}_{\text{P.G.F.}} + \underbrace{\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)}_{\text{viscous}}$$

$$\frac{\partial v}{\partial t} = \quad '' \quad\quad '' \quad + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\frac{\partial w}{\partial t} = \quad '' \quad\quad '' \quad + \frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

$(-g)$

general $$\frac{\partial \vec{V}}{\partial t} = \underbrace{-\vec{V}\cdot\nabla\vec{V}}_{\text{I}} \underbrace{- \frac{1}{\rho}\nabla p}_{\text{PGF}} + \underbrace{\frac{\mu}{\rho}\nabla^2\vec{V}}_{\text{Visc}}$$

Steady
$$0 = \text{I} + \underline{\underline{\text{PGF}}} + \text{Visc}$$

"Turbulent"

Reynolds # $\approx \frac{|\text{I}|}{|\text{Visc}|}$

$\underline{Re}$

High Re:
$|\text{I}| \gg |\text{Visc}|$

primary balance
$\text{I} \sim \text{PGF}$

"Laminar"

Low Re:
$|\text{I}| \ll |\text{Visc}|$
$\text{PGF} \sim \text{Visc}$

$$0 \approx -\vec{v}\cdot\nabla\vec{v} - \frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2\vec{v}$$

$$\frac{\mu}{\rho} \equiv \nu$$

$$Re \sim \frac{|-\vec{v}\cdot\nabla\vec{v}|}{|\nu\nabla^2\vec{v}|} \sim \frac{UL}{\nu}$$

$L \quad U$