Lecture 8

9/16

Continue from Lecture 7, want to explain:

in Fluent $\left\{ \right.$

* p-<u>based</u> vs. ρ-<u>based</u> solver
* <u>turbulence</u> vs. <u>laminar</u> model
* scaled residual
* <u>steady</u> vs. <u>transient</u> solution

Recap (Lec 7) : Navier-Stokes eq. $\underbrace{\text{(momentum eq.)}}_{\text{Viscous}}$

$$\left\{\begin{aligned}
\frac{\partial u}{\partial t} &= \underbrace{-u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z}}_{\text{Inertial}} \underbrace{-\frac{1}{\rho}\frac{\partial p}{\partial x}}_{\text{PGF}} + \overbrace{\nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)}^{\text{Viscous}} \\
\frac{\partial v}{\partial t} &= \ldots\ldots \\
\frac{\partial w}{\partial t} &= \ldots\ldots\ (-g)
\end{aligned}\right.$$

kin. vis $\equiv \dfrac{\mu}{\rho}$

μ: dyn. vis.

vector form

$$\frac{\partial \vec{V}}{\partial t} = \underbrace{-\vec{V}\cdot\nabla\vec{V}}_{\text{I}} \underbrace{-\frac{1}{\rho}\nabla p}_{\text{PGF}} + \underbrace{\nu\nabla^2\vec{V}}_{\text{Visc}}$$

$$\frac{\partial \vec{v}}{\partial t} = \underbrace{-\vec{v}\cdot\nabla\vec{v}}_{I} \underbrace{- \frac{1}{\rho}\nabla p}_{PGF} + \underbrace{\nu \nabla^2 \vec{v}}_{V}$$

$$Re \sim \frac{|I|}{|V|} \sim \frac{\frac{U^2}{L}}{\nu\left(\frac{U}{L^2}\right)} \sim \frac{UL}{\nu}$$

$$\underbrace{-u\frac{\partial u}{\partial x}}_{\frac{U^2}{L}} - v\frac{\partial u}{\partial y} - \ldots \text{etc.}$$

$|I| \gg |V|$ large Re

primary balance $I \sim PGF$

turbulence

$|V| \gg |I|$

laminar small Re

$V \sim PGF$

Fluent

large $Re \rightarrow$ turbulence model

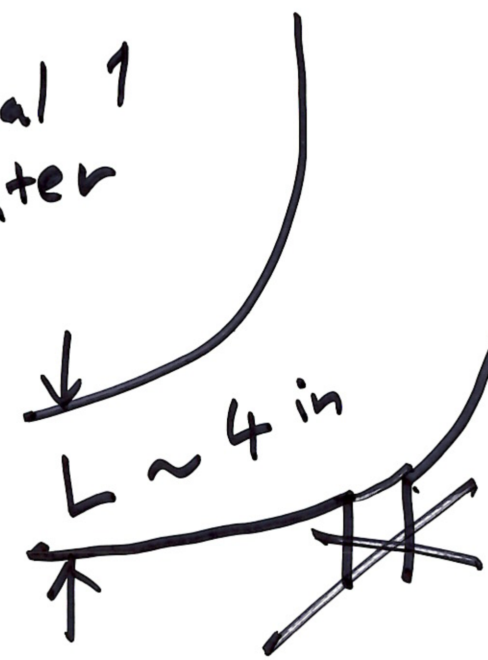
small $Re \rightarrow$ laminar

~~HW 1~~
Tutorial 1
water

$$\nu_{water} = \frac{\mu}{\rho} \sim 10^{-6}$$

$\mu \approx 1 \times 10^{-3}$ Pa·m

ρ — 1000 kg/m3



$U \rightarrow$

0.4 $\frac{m}{s}$

0.04 $\frac{m}{s}$

0.04 in

4 in ~ 0.1 m

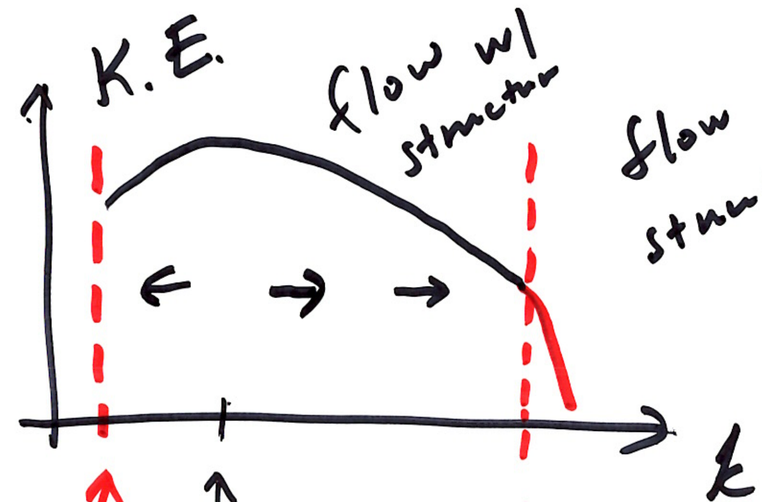
$$Re \sim \frac{UL}{\nu}$$

$$\sim \frac{(0.4)\cdot(0.1)}{10^{-6}} \sim 40000$$

statistical equilibrium

$k = \frac{2\pi}{L}$

ω^2



large L

energy injection

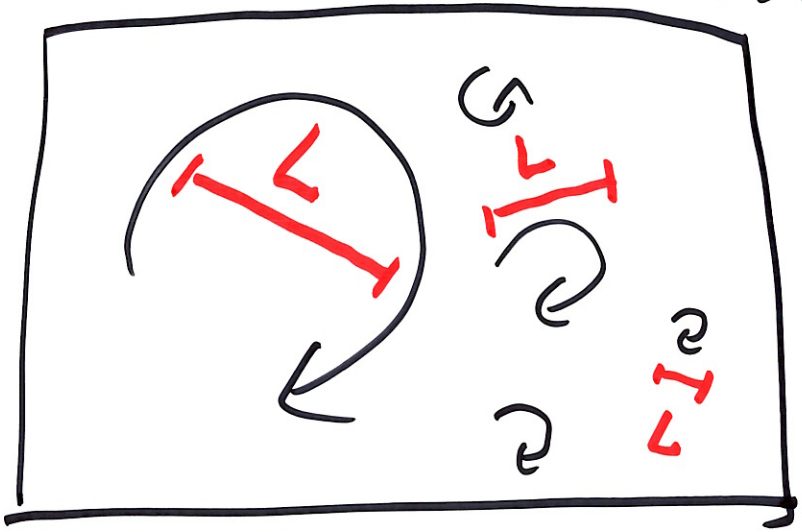
small L

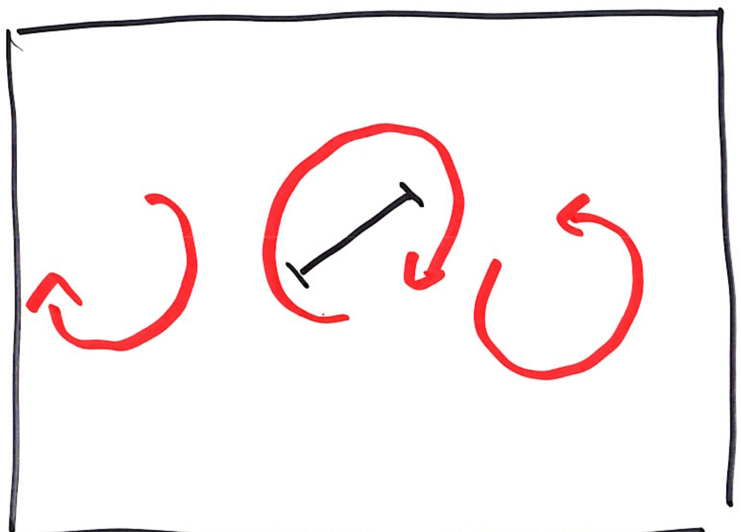
limited by finiteness of domain ("condensation" into coherent structure ?.)

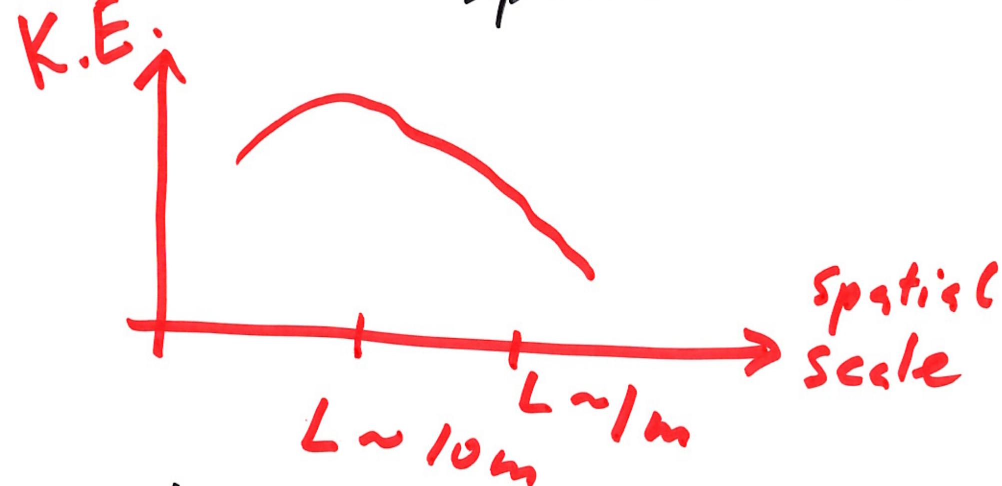
viscous effect becomes ~~important~~ important

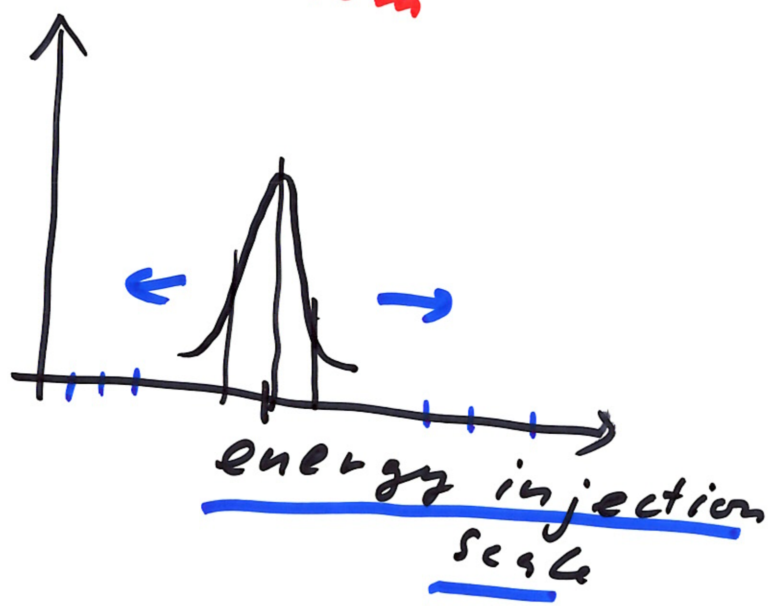
"Kolmogorov microscale"

(I) : nonlinear self interaction → produce a "spectum" of different spatial scales









$$u \sim \sin(\underline{\underline{2}}x) + \sin(\underline{\underline{3}}x) \qquad k \sim \frac{2\pi}{L}$$

$$u \cdot u$$

trig identity

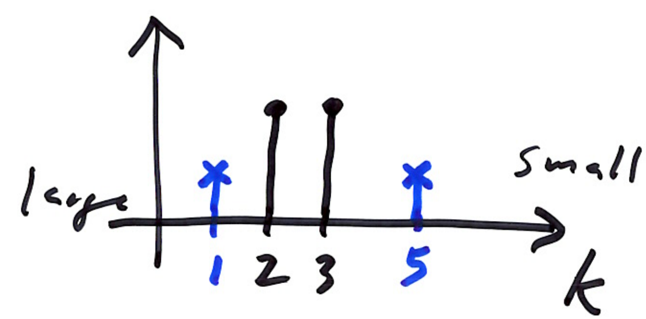
$[\sin(2x)]^2$

$[\sin(3x)]^2$

⋮

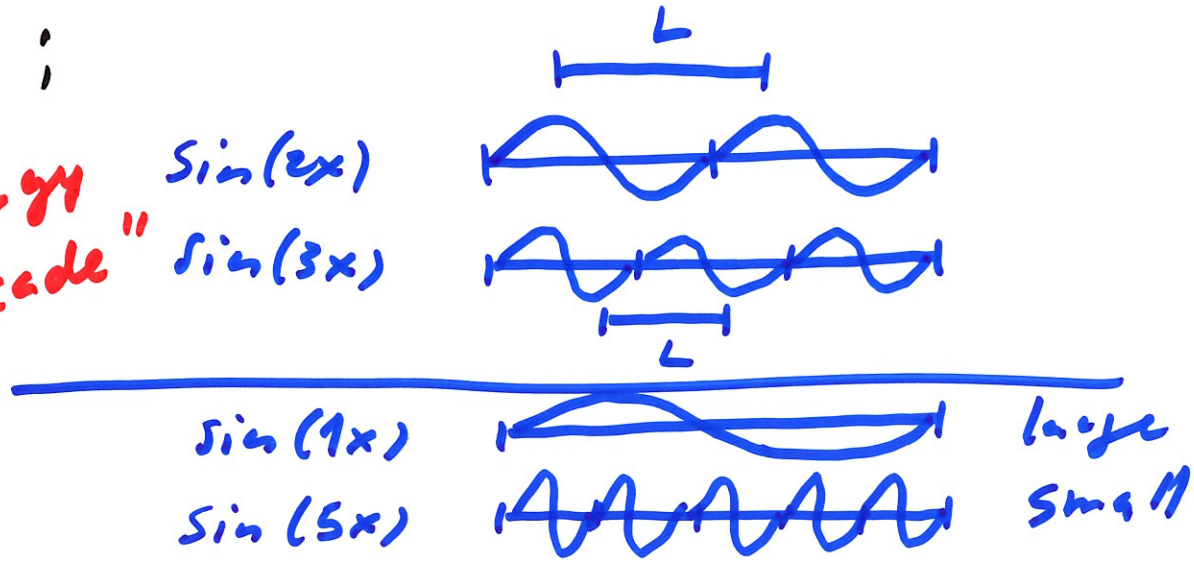
$$\sin(2x)\sin(3x)$$
$$= \frac{1}{2}\cos((3-2)x) - \frac{1}{2}\cos((3+2)x)$$
$$= \frac{1}{2}\cos(1x) - \frac{1}{2}\cos(5x)$$

domain



large k $\Updownarrow$ small L

"energy cascade"



Viscous term

$$\frac{\partial u}{\partial t} = \ldots \quad + \underline{\nu \nabla^2 u}$$

$$-k^2 \sin(kx)$$

$$u \sim \sum a_k \boxed{\underline{\sin(kx)}}''$$

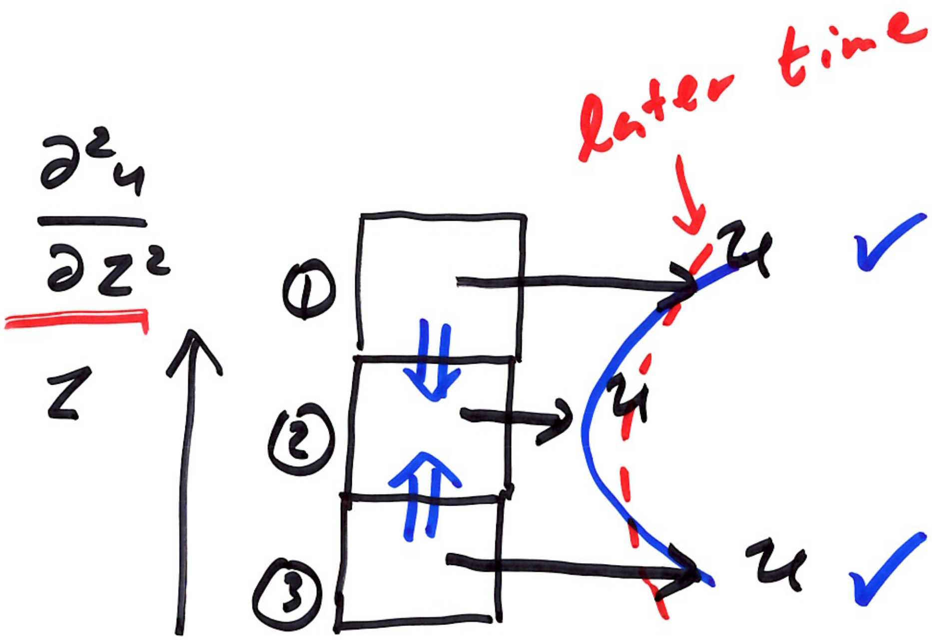
$$+ b_n \cos(kx)$$

$$\nabla^2 u \rightarrow \sum - a_k \underline{\underline{k^2}} \sin(kx) + - b_k k^2 \cos(kx)$$

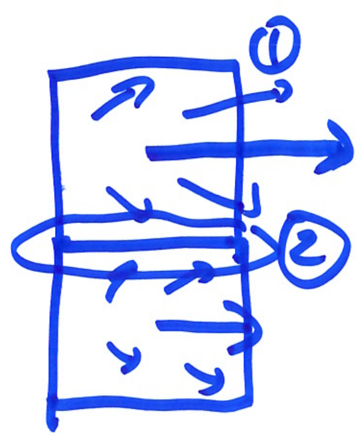
$\nu \nabla^2 u$ becomes very large at $\underline{\text{small } L}$ $\qquad k \equiv \dfrac{2\pi}{L}$

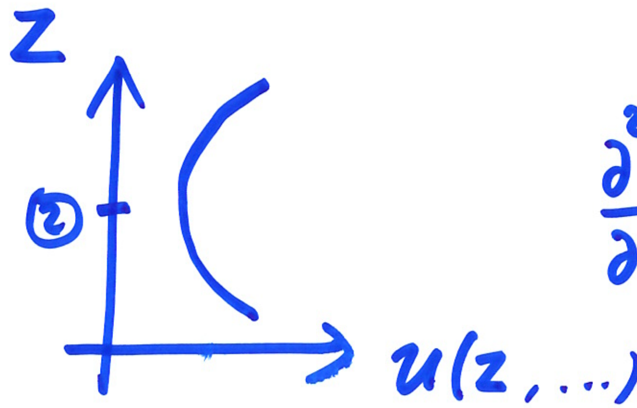
$$\cancel{\text{dt}} \quad \frac{da_k}{dt} = - a_k \cdot k^2$$

$$\underline{\frac{\partial u}{\partial t}} = \cdots\cdots + \nu \underline{\frac{\partial^2 u}{\partial z^2}}$$

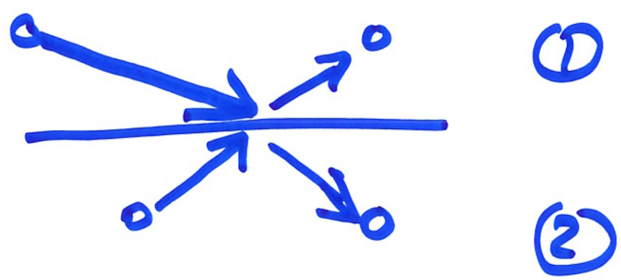


Lec 7



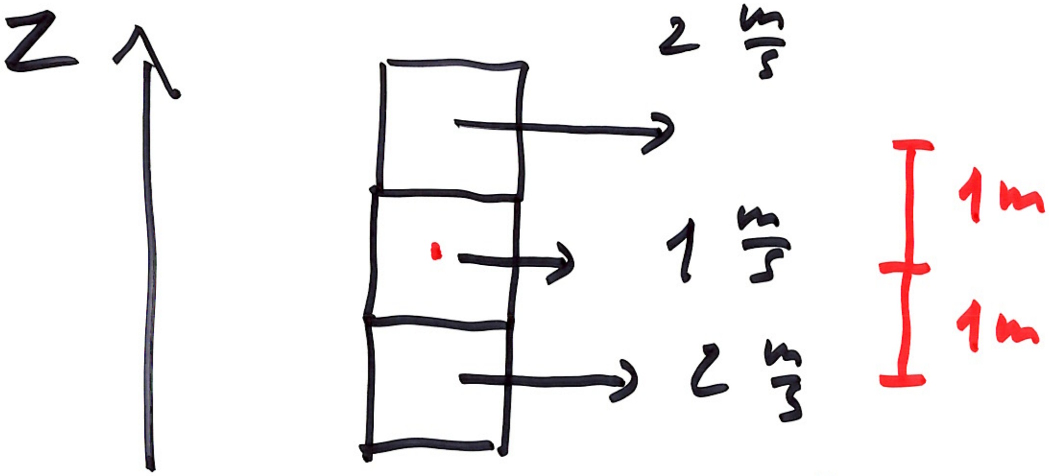


$\frac{\partial^2 u}{\partial z^2} > 0$ at local min

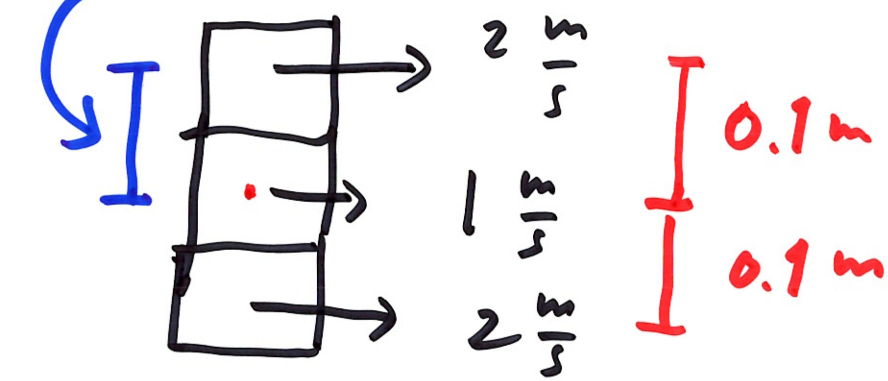


Case 1

u



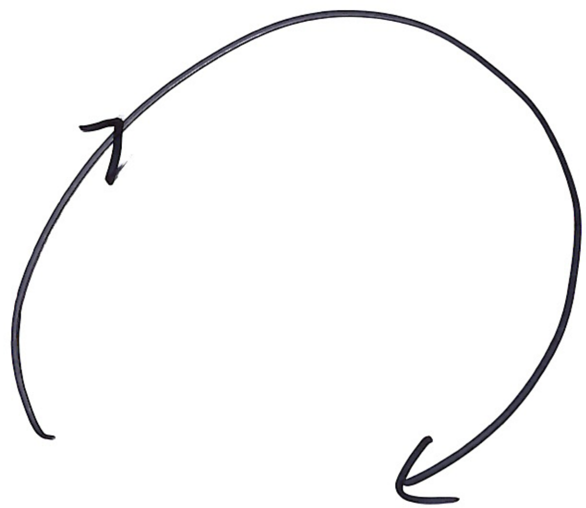
U difference in vel.



$$\frac{\partial^2 u}{\partial z^2} < \frac{\partial^2 u}{\partial z^2} \quad 100 \text{ times}$$

$$\nu \frac{\partial^2 u}{\partial z^2} \sim \nu \cdot \frac{U}{L^2} \qquad \frac{\partial}{\partial z}\left(\frac{\partial}{\partial z}\right) \qquad \hat{z} = 0.1\, z$$

small L $\rightarrow$ large visc. term



"structure" is killed

influend by visc.

large eddy

U

L large :
almost inviscid
!

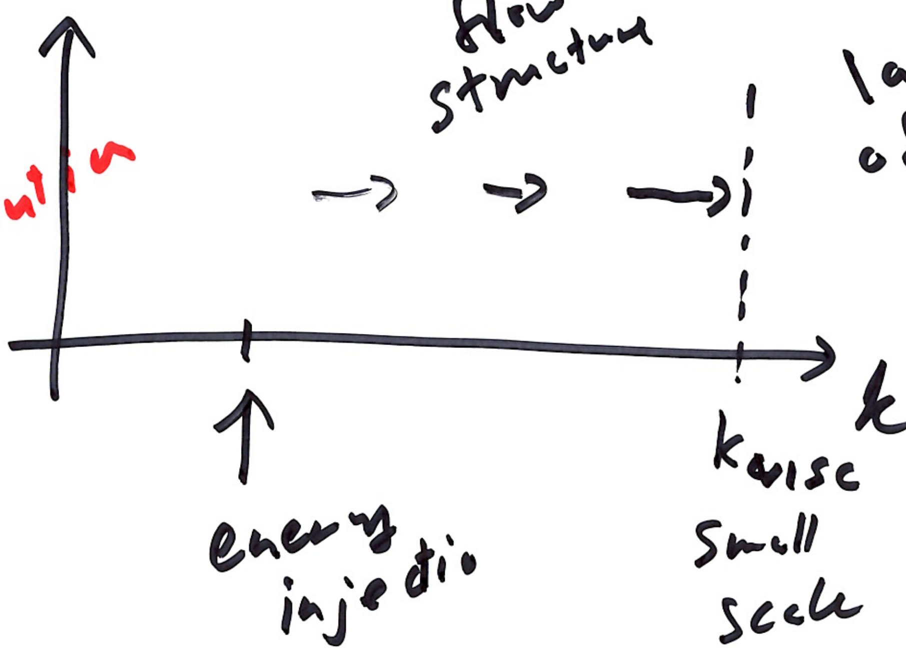
at Kolmogorov scale

eddies do not survive 1 turn-over time before being damped by visc.

high Re flow

Requires higher mesh resolution !!

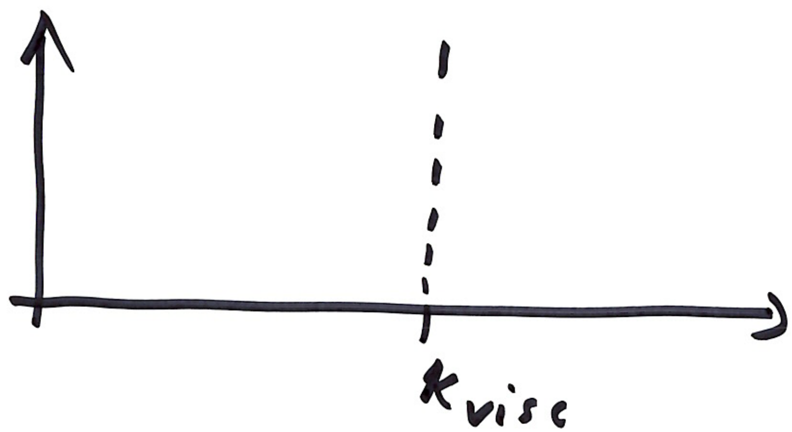
large L

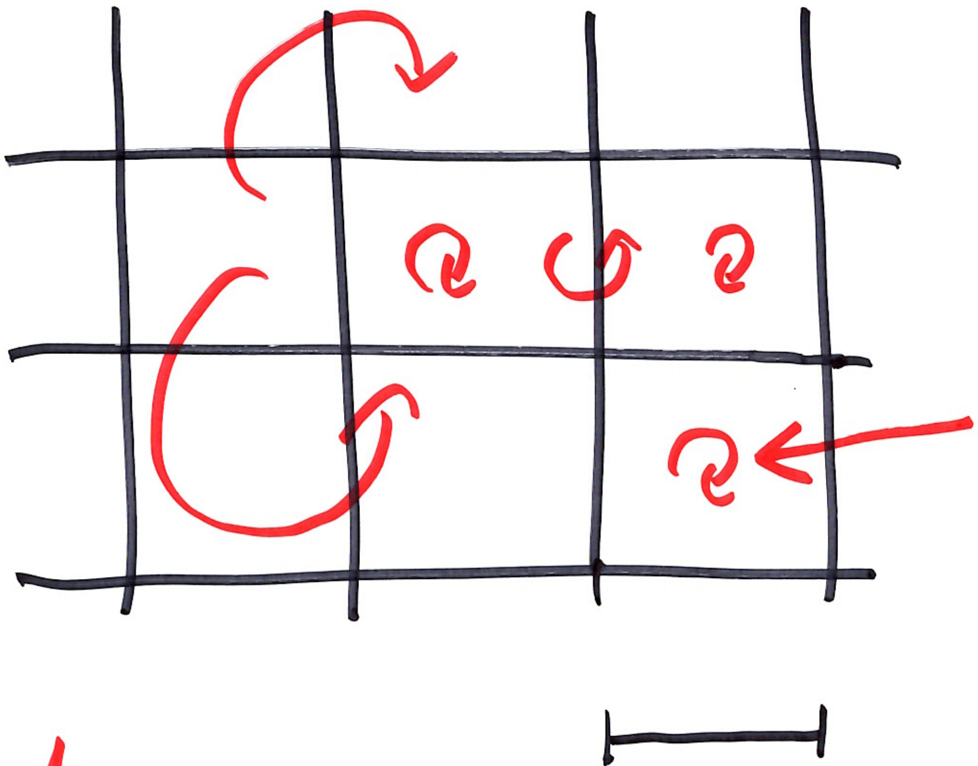


lack of flow struct

small L

low Re flow





mesh

unresolved by the mesh !

sub-grid scale structures

SGS

ℓ element size

High Re flow (turbulence)

turbulence model (e.g. k-ε model)

emulate the total effect of SGS on the resolved flow at the nodes

k-ε model extra variables to compute

k field vars just like u, v, w

ε $T \ldots$

☑ k-ε model in fluent, 3-D, energy ☑

☑ Laminar

✓ $\frac{\partial u}{\partial t} = \ldots$

✓ $\frac{\partial v}{\partial t} = \ldots$

✓ $\frac{\partial w}{\partial t} = \ldots$

✓ continuity

✓ energy

X

✓ $\frac{\partial k}{\partial t} = \ldots$

✓ $\frac{\partial \varepsilon}{\partial t} = \ldots$

residual plot

p-based vs. ρ-based solver

$$\frac{\partial \vec{V}}{\partial t} = -\vec{V}\cdot\nabla\vec{V} - \frac{1}{\rho}\nabla p + \nu\nabla^2\vec{V} \qquad \text{——— (1)}$$

Continuity

incomp. flow $\quad 0 = \nabla\cdot\vec{V} \qquad \text{——— (2)}$

$\boxed{\text{compressible flow}}$ $\quad \dfrac{\partial\rho}{\partial t} = -\nabla\cdot(\rho\vec{V})$

$\dfrac{d\rho}{dt} = 0$

need to compute ρ

ρ-based

in compressible

$\nabla \cdot ①$:

$$\frac{\partial\, \cancel{\nabla \cdot \vec{v}}}{\partial t} = \underbrace{\nabla \cdot \left(-\vec{v} \cdot \nabla \vec{v} + \nu \nabla^2 \vec{v} \right)} - \underbrace{\frac{1}{\rho} \nabla \cdot \nabla p}_{\frac{1}{\rho} \nabla^2 p}$$

② 0

$$\Downarrow$$

$$\nabla^2 p = -\rho\, \nabla \cdot \left(-\vec{v} \cdot \nabla \vec{v} + \nu \nabla^2 \vec{v} \right)$$

HW 1 $\rho = \text{const}$

$$\underline{\nabla^2 p = S(\vec{v})}$$

Poisson eq.

p-based solver

solve p from $\vec{v}$ in order to maintain conservation of mass!