Lecture 9

9/21

Project 1: will be posted very soon!

Today: Background

Thu + next Tue: Demos

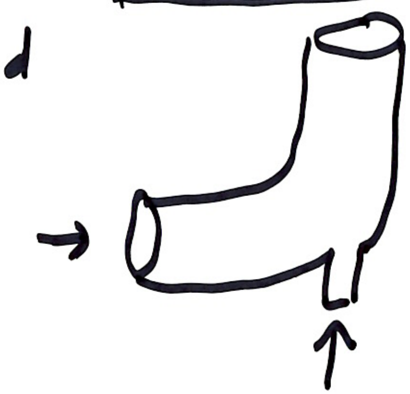
New things in Proj 1 beyond HW1:

Tutorial #1 } artificial $\boxed{\rho = \text{const}}$

ignored

① physically $\rho \equiv \rho(T)$

Thermal convection by buoyancy effect

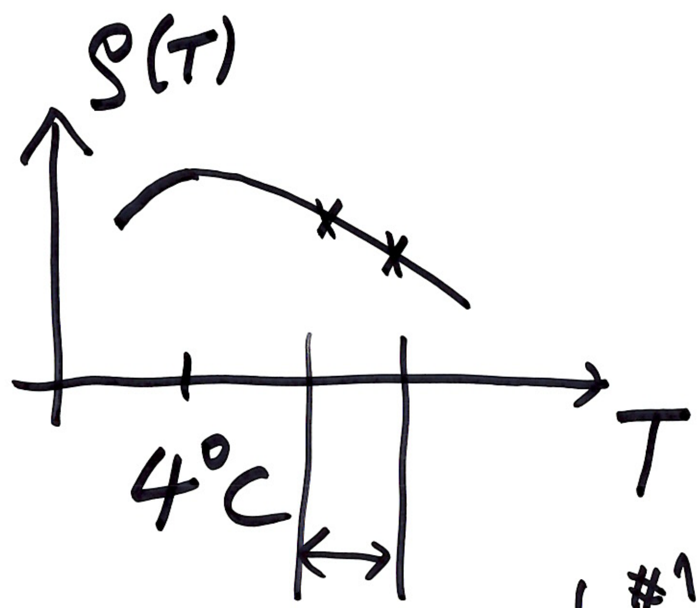


② ρ-based solver for compressible flow

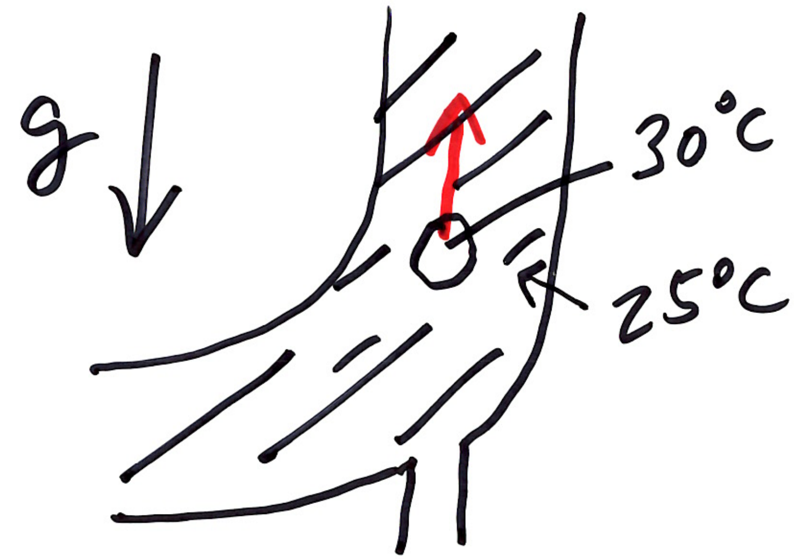
③ Transient solution

(④ Misc. e.g. convergence criterion/monitor etc)

ρ Water



~~Tu~~ Tutorial #1



" * Want to emulate the buoyancy effect and yet stay within the framework of ~~in~~ incompressible/p-based solver "

"Boussinesq approximation"

z-momentum w

E $$\underline{\frac{\partial w}{\partial t}} = \underline{-u\frac{\partial w}{\partial x} - v\frac{\partial w}{\partial y} - w\frac{\partial w}{\partial z}} - \frac{1}{\rho}\frac{\partial p}{\partial z} + \underbrace{\nu(\text{visc.})}_{\text{ignore it for now}} - g$$

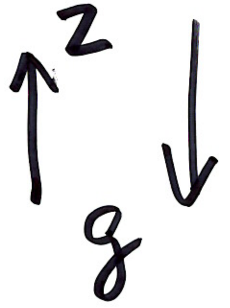
$\downarrow$

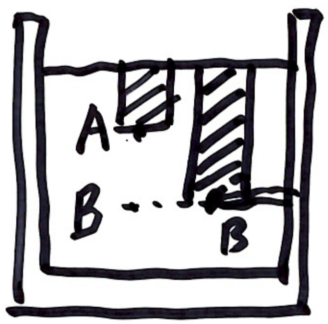
L $$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

The case with $\frac{dw}{dt} = 0$:

$$\underline{0 = \frac{1}{\rho}\frac{\partial p}{\partial z} - g}$$

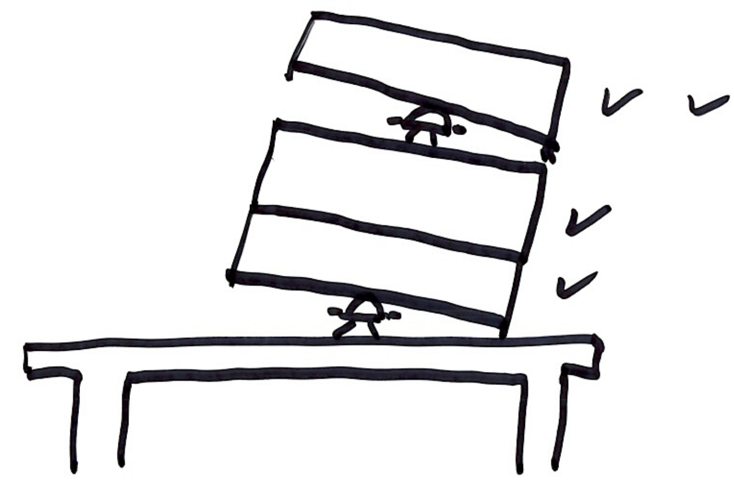
hydrostatic balance

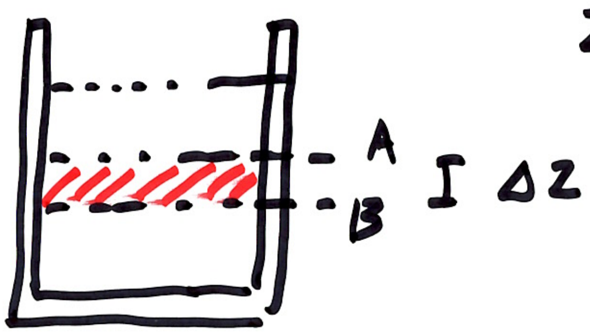




$p_B > p_A$

$z_B < z_A$



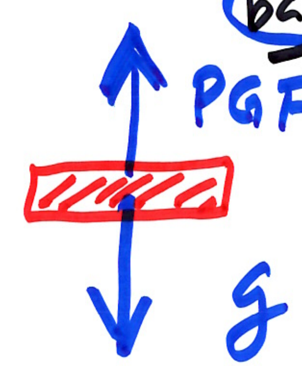


$\text{mass} = \rho \cdot \underline{\Delta z \cdot A}$

$g(\downarrow) = \underbrace{\rho g \Delta z}_{\sim -\Delta p} \cdot A$

hydro "static"

"balance"



$\rho g \Delta z \sim -\Delta p$

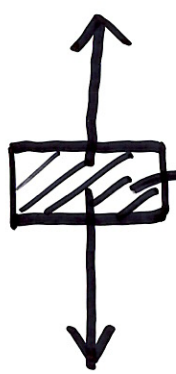
$$\boxed{\frac{\Delta p}{\Delta z} = -\rho g}$$

✓

$$\underbrace{\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g}_{\text{imbalance}} \Rightarrow \frac{dw}{dt} \text{ under buoyancy force}$$

hydrostatic

$\underline{p(z)}$



$\frac{dp}{dz}$

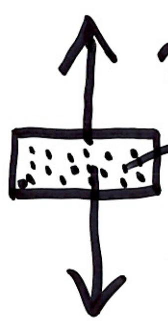
environment

$-\rho g$

parcel

ρ_{PAR}

ρ_{ENV}



$\approx -\rho_{ENV}\, g$

$-\rho_{PAR}\, g$

$\rho_{PAR} \neq \rho_{ENV}$

$$\underline{-\rho_{ENV}\, g} = \frac{dp_E}{dz}$$

$$\rho_{PAR}\frac{dw}{dt} = -\frac{dp}{dz} - \rho g \approx -(\rho_{PAR} - \rho_{ENV})\, g$$

$$\frac{dw}{dt} = -\left(\frac{\rho_{PAR} - \rho_{ENV}}{\rho_{PAR}}\right) g$$

Achemides' principle

ENV PAR

$$\frac{dw}{dt} = -\underbrace{\left(\frac{\Delta\rho}{\rho}\right) g}_{\text{"reduced gravity"}}$$

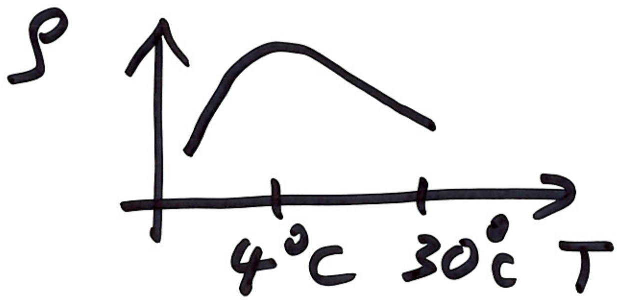
baseball

ρ_{BAS}

air

ρ_{AIR}

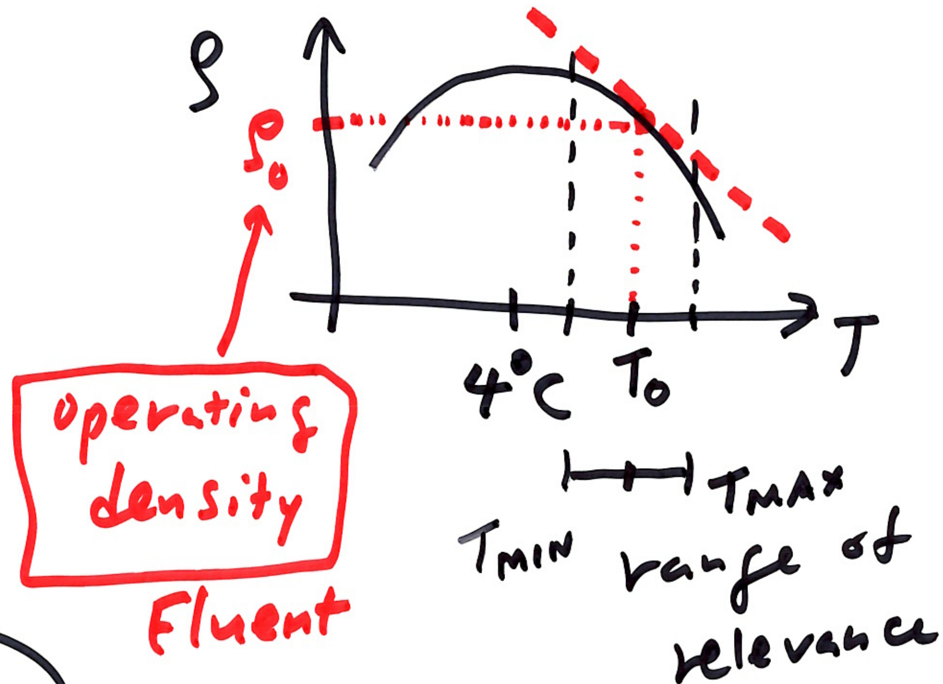
water $\rho(T)$



$$\frac{dw}{dt} = -\left(\frac{\rho_{BAS} - \rho_{AIR}}{\rho_{BAS}}\right) g$$

$$\approx -g$$

$\rho \equiv \rho(T)$



$$\boxed{\frac{dw}{dt} = -\left(\frac{\rho - \rho_0}{\rho_0}\right) g}$$

Boussinesq approx.

Say $T_0 \equiv \dfrac{T_{MAX} - T_{MIN}}{2}$

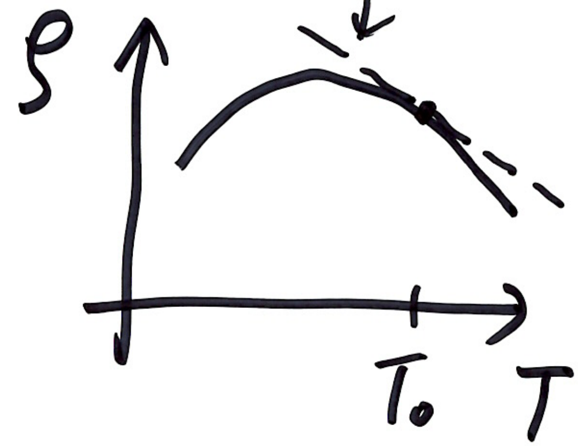
Fluent:

✓ $\boxed{\text{Operating Temperature}}$

$$\rho(T) = \underbrace{\rho(T_0)}_{\rho_0} + \left(\frac{d\rho}{dT}\right)_{T_0}(T-T_0) + \cancel{\left(\frac{d^2\rho}{dT^2}\right)_{T_0}\frac{(T-T_0)^2}{2}} + \cdots$$

$$\rho - \rho_0 \approx \left(\frac{d\rho}{dT}\right)_{T_0}(T-T_0)$$

The "slope"



$$\frac{\rho-\rho_0}{\rho_0} \approx \frac{1}{\rho_0}\left(\frac{d\rho}{dT}\right)_{T_0}(T-T_0)$$

replace it by

$\boxed{\text{Thermal expansion coefficient}}$ in Fluent

$$\frac{\Delta\alpha}{\alpha}\cdot\frac{1}{\Delta T}$$

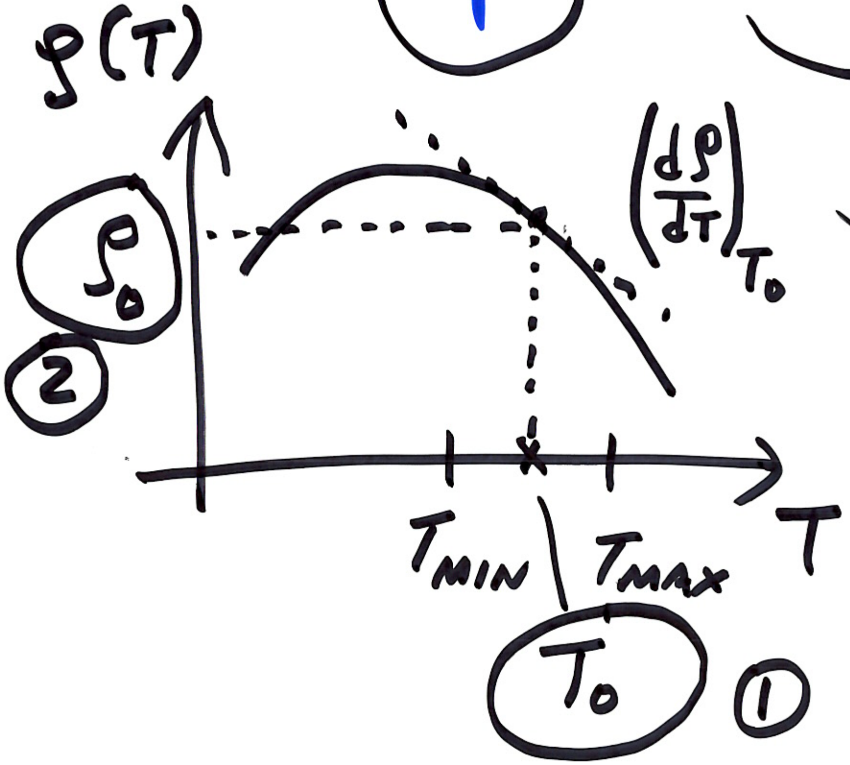
$$\beta \equiv \frac{1}{\alpha}\frac{d\alpha}{dT} \qquad \alpha \equiv \frac{1}{\rho} \text{ specific volume}$$

$$\alpha = \frac{1}{\rho} = \rho^{-1}$$

$$\frac{1}{\alpha}\frac{d\alpha}{dT} = -\frac{1}{\rho}\frac{d\rho}{dT}$$

3

$$\beta = -\frac{1}{\rho_0}\left(\frac{d\rho}{dT}\right)_{T_0}$$



$$\boxed{\frac{dw}{dt} = \beta(T - T_0)g}$$

✓

"Revised" z-momentum eq.

Boussinesq:

$$\frac{\partial w}{\partial t} = -u\frac{\partial w}{\partial x} - v\frac{\partial w}{\partial y} - w\frac{\partial w}{\partial z} + \nu(\text{visc.}) + \beta(T-T_0)g$$

Continuity eq. $\rightarrow$ keep as incompressible

(ρ-based)

Energy eq: (1st law of thermo.)

$$đQ = dU + p\,dV$$
$$= c_v\,dT + p\,d\underline{\underline{V}}$$

$$đq = c_v\,dT + p\,d\underline{\underline{\alpha}} \qquad \alpha \equiv \frac{1}{\rho}$$
$$= c_v\,dT + p\,d\left(\frac{1}{\rho}\right)$$

adiabatic $đq = 0$

$$0 = c_v\,dT + p\,d\left(\frac{1}{\rho}\right) \qquad L$$

$$0 = c_v\left(\frac{\partial T}{\partial t} + \vec{v}\cdot\nabla T\right) + p\left[\frac{\partial\left(\frac{1}{\rho}\right)}{\partial t} + \vec{v}\cdot\nabla\left(\frac{1}{\rho}\right)\right] \qquad E$$

$$0 = C_v\, dT + p\, d\left(\frac{1}{\rho}\right)$$

Boussinesq

$$\cancel{\frac{d\rho}{\rho}} \quad \frac{1}{\rho_0}\left(\frac{d\rho}{dT}\right) \to -\beta$$

$$\longrightarrow \quad \underline{d\rho} \approx -\beta\,\rho_0 \cdot \underline{dT}$$

$$0 = C_v\, dT - p\,\frac{d\rho}{\rho^2} \approx \frac{-\beta\rho_0\, dT}{\rho_0^2} \approx \frac{-\beta\, dT}{\rho_0}$$

$$\approx C_v\, dT + \beta p \rho_0^{-1}\, dT$$

$$\approx \left(C_v + \beta p \rho_0^{-1}\right) \underline{\underline{dT}}$$