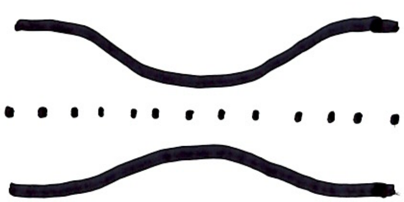
Lecture 15

10/14

* Project 1 is due Friday. <u>Canvas</u>
* "Statement of Collaboration" is required. *

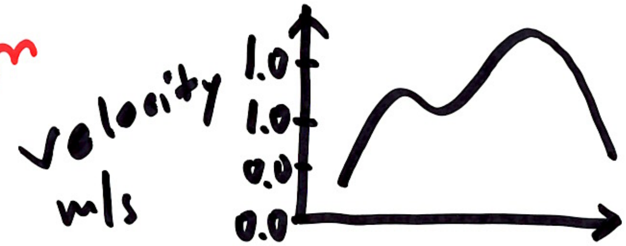
* Figures should be clearly labelled *

line plot
same deal as Task 2 of HW1

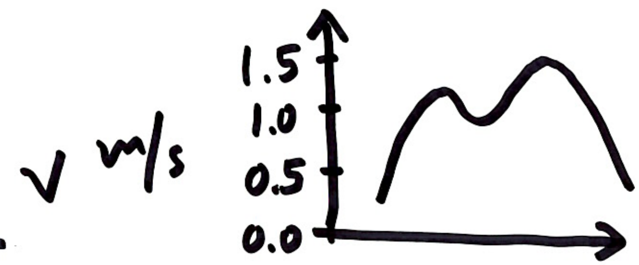


← new surface

Deduction



✓



* Tip on <u>Task 3</u>

$M \equiv \frac{u}{c}$ ← speed of sound

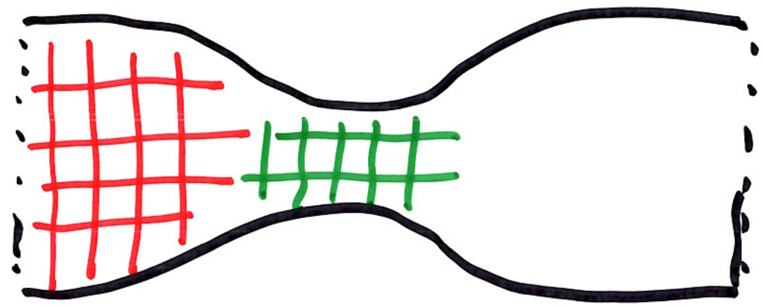
"<u>Mach number</u>" → Fluent has it

$\boxed{\text{Velocity}}$

- X-vel
- Y-vel
- ⋮
- ✓ → Mach number

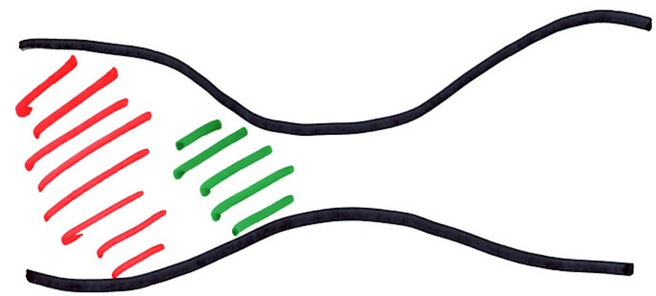
Task 3 (2-D)

contour plot



<u>filled</u> contour plot

~~ue~~ uncheck
all surfaces



Project 2 — two-phase flow (multi-phase)

flow w/ interface

HW 1: uniform fluid

1 fluid

const ρ

Proj 1 Task 3a

more variation of ρ but 1 fluid

Proj ~~2~~ 1 / Task 1

1 fluid

$\rho \equiv \rho(T)$

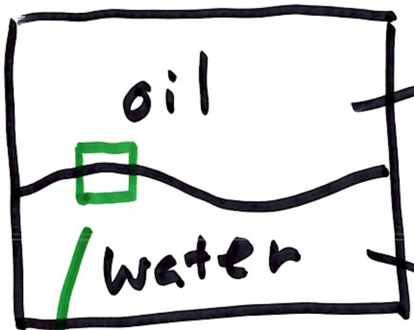
$10^\circ C$ → $40^\circ C$ T

$|\nabla \rho|$ not too high

Boussinesq. would work if $\frac{\Delta \rho}{\rho_0} \ll 1$

$$\frac{\Delta \rho}{\rho_0} \approx \frac{\rho_{10c} - \rho_{40c}}{\rho_{25c}} \overset{?}{<} 1\%$$

small



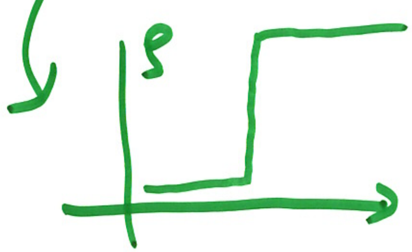
$\rho_{oil} \sim 800 \ kg/m^3$

$\rho_w \approx 1000 \ kg/m^3$

$\nabla \rho$ at interface extremely large !!

$$\frac{\Delta \rho}{\bar{\rho}} \sim 20\%$$

$$\frac{\Delta \rho}{\bar{\rho}} \sim \text{very large}$$



$$\nabla \rho = \frac{\partial \rho}{\partial x} \hat{i} + \frac{\partial \rho}{\partial y} \hat{j} + \frac{\partial \rho}{\partial z} \hat{k}$$

Ex: Continuity eq.

$$\frac{\partial \rho}{\partial t} = -\vec{V}\cdot\underline{\underline{\nabla\rho}} - \rho\,\nabla\cdot\vec{V}$$

Viscous term in x momentum eq:

If μ = const. ρ = const

$$\nu \equiv \frac{\mu}{\rho}$$

$$\underline{VISC} = \nu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \ldots\right]$$

moreover ...

two fluids

viscosity $\mu_1 \neq \mu_2$

$K_1 \neq K_2$

...

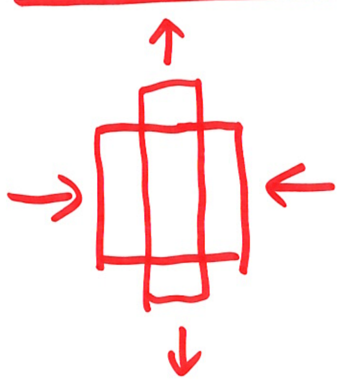
$$\underline{VISC} = \frac{1}{\rho}\left[\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \ldots\right]$$

$\nabla\mu$

Consider: 2 fluids, each individually behave like incompressible w/ const ρ

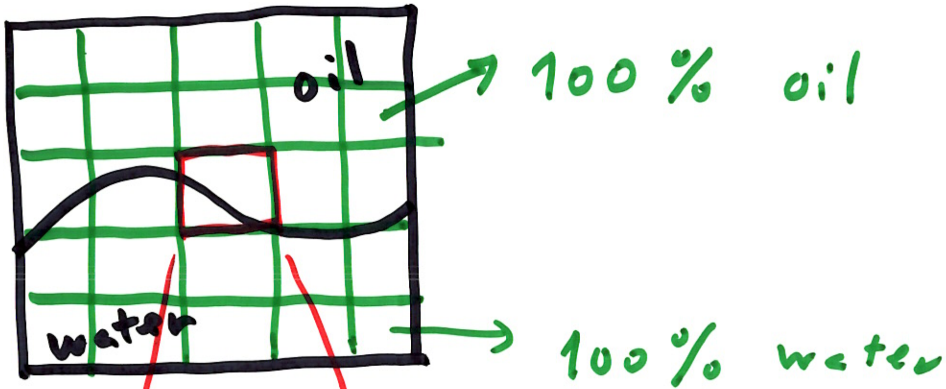
$\rho_{oil} = \text{const} = 800$

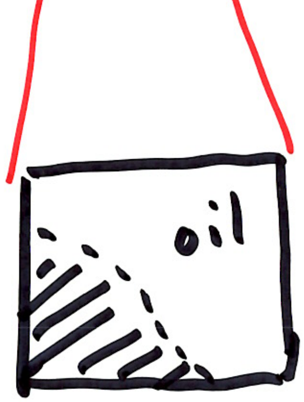
$\rho_w = \text{const} = 1000$



$$\frac{d\rho}{dt} = 0$$

fluid system as a whole is still <u>incompressible</u>





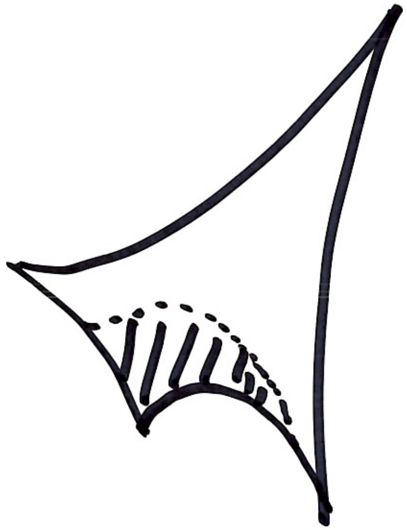
70 % oil $\quad VF_o = 0.7$

30 % water $\quad VF_w = 0.3$

(by volume)

volume fraction (VF) $\quad \underline{VF_1 + VF_2 = 1}$

suffices to compute one of them



C_1

VF_1 is conserved

VF_2 is conserved

C_2

for the parcel

$C_1 + C_2 = 1$

$0 \le C_1 \le 1$

$0 \le C_2 \le 1$

$$\frac{d\rho}{dt} = 0$$

$$\frac{d(VF_1)}{dt} = 0$$

<u>Lagrangian</u>

$$\frac{d(VF_2)}{dt} = 0$$

use it to replace continuity eq.

$$\frac{dC_1}{dt} = 0 \longrightarrow \boxed{\frac{\partial C_1}{\partial t} = -\vec{V} \cdot \nabla C_1}$$

Eulerian

Volume of Fluid method

"VOF" — Fluent

✓ Multiphase

↳ "VOF"

$$\frac{\partial C_1}{\partial t} = -\vec{V} \cdot \nabla C_1$$

numerical stability

Fluent

Explicit

Implicit

Each fluid parcel has only 1 value of C_1 (and $C_2 = 1 - C_1$)

ONLY 1 velocity for the mixture

ONLY 1 pressure for mixture

ONLY 1 momentum eq. for the mixture

~~for~~ at every time step:

$$\boxed{\frac{\partial \vec{V}}{\partial t} = -\vec{V}\cdot\nabla\vec{V} - \frac{1}{\rho}\nabla p + \text{VISC} \quad \text{momentum eq.}}$$

$\vec{V}$

use it

$$\frac{\partial C_1}{\partial t} = -\vec{V}\cdot\nabla C_1$$

$$C_1 \longrightarrow \underline{\underset{\text{total}}{\rho}} = C_1 \underset{\text{const}}{\underline{\underline{\rho_1}}} + C_2 \underset{\text{const}}{\underline{\underline{\rho_2}}}$$

$C_2 = 1 - C_1$

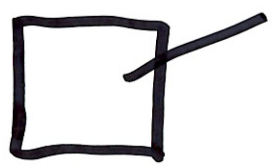
3-phase ?

phase 1 — water

phase 2 — oil

phase 3 — air

C_1 is field just like u, v, w, T, p



$$C_1 = 0.5$$
$$C_2 = 0.3$$
$$C_3 = 0.2$$

$$\underline{\underline{C_1}} + \underline{\underline{C_2}} + C_3 = 1$$

Need to solve

$$\frac{\partial C_1}{\partial t} = -\vec{V} \cdot \nabla C_1$$

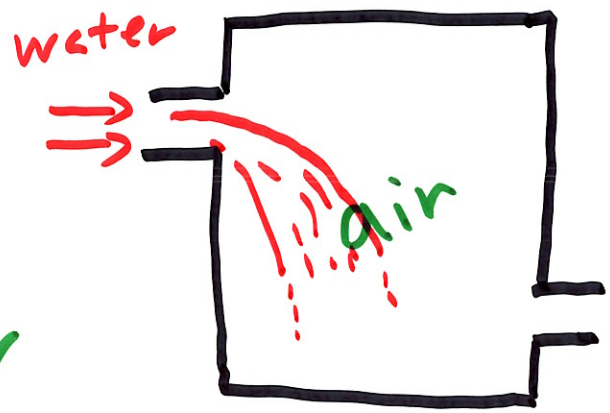
$$\frac{\partial C_2}{\partial t} = -\vec{V} \cdot \nabla C_2$$

$$\rho = C_1 \rho_1 + C_2 \rho_2 + C_3 \rho_3$$

Proj 2

Under gravity

g

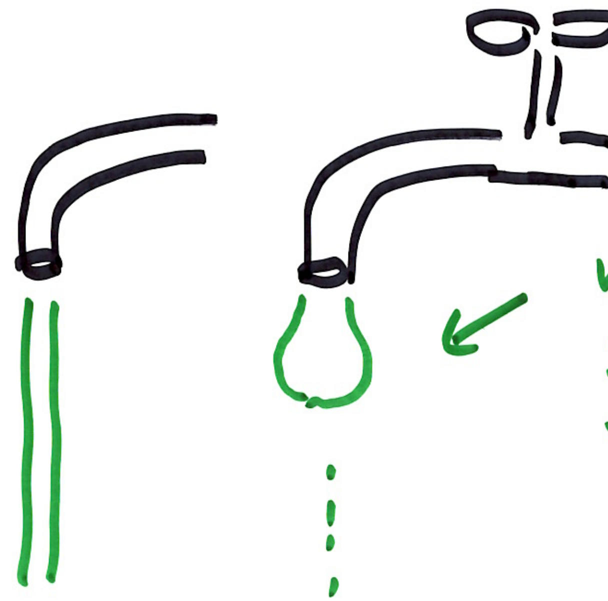


transient

$t=0$

additional physical process at interface (surface tension)

btwn liquid & gas



without surface tension

with surface tension

Proj 2 — All transient solution

Tutorial #3
(Useless!)

Please come to class
for the correct demo