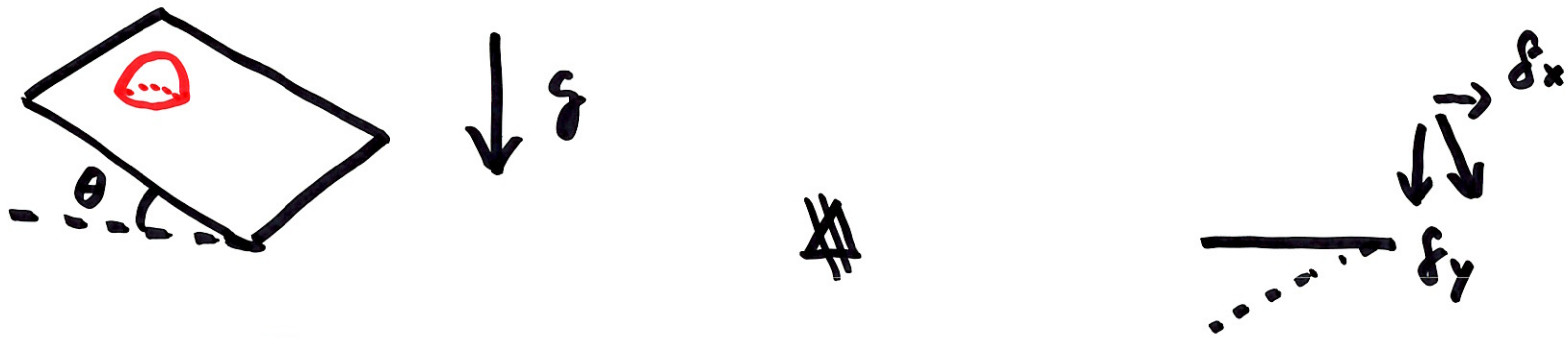
Lecture 19

10/28

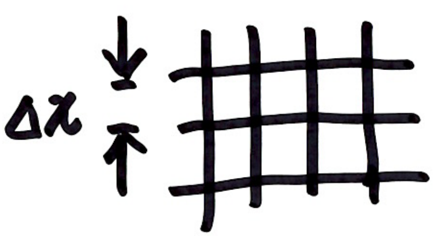
Proj 2 Task 3



Recap (Lec 18)

transient solution

<u>Δx</u>



<u>Δt</u>

VOF

- ☐ Explicit → numerical Stability
- ☐ Implicit

always numerically stable → No implications on accuracy

roughly

max velocity

$$C \sim \frac{U \Delta t}{\Delta x}$$

Courant #

$$C > 1$$

numerical instability — Δx, Δt

↓ remedies to stabilize the run

either decrease Δt

OR increase Δx

coarsening the mesh

General guidelines Δx Δt Proj 2

✓ ① IF Explicit ⟵ avoid $C > 1$

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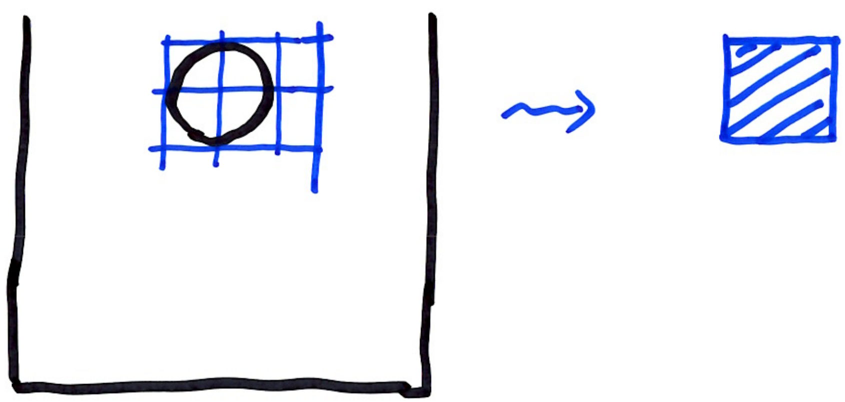
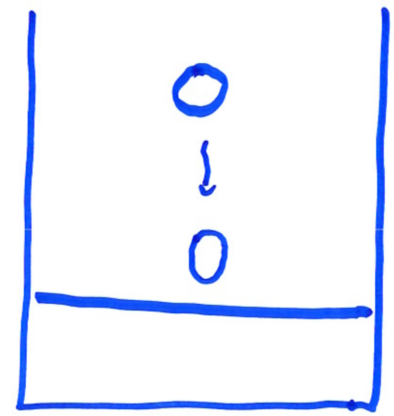
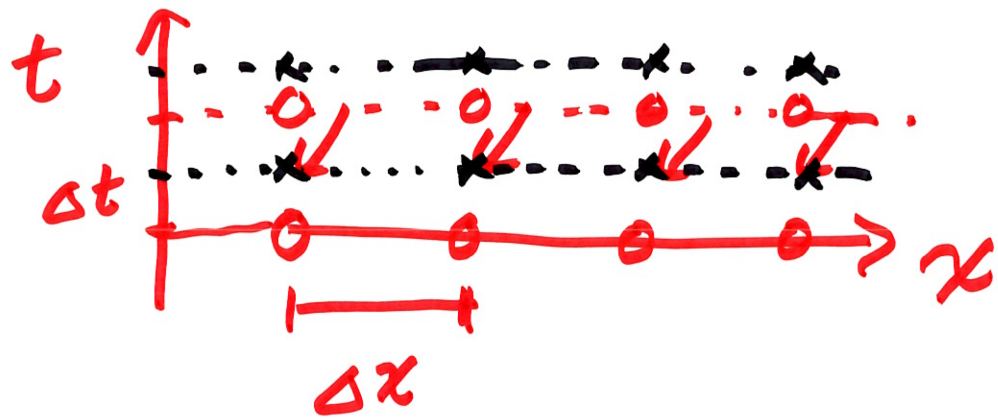
* Δx is small enough to resolve critical structures of the flow, up to $t = t_{target}$





Task 2

Note: Smaller C does NOT imply a higher level of accuracy

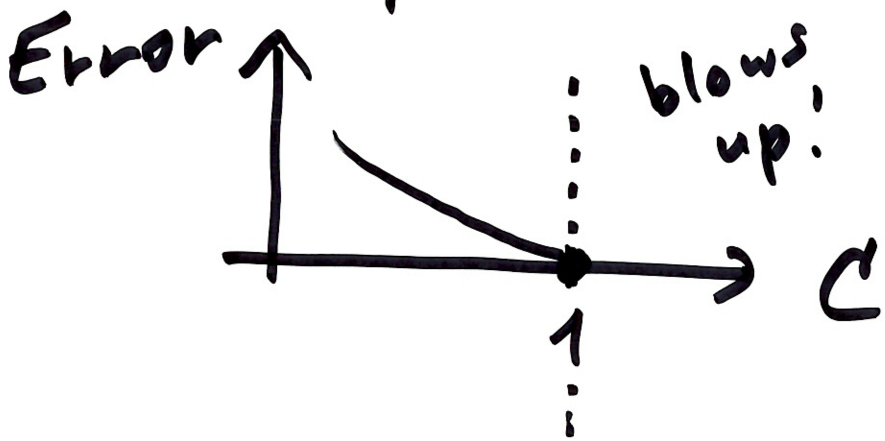


Ex: $u(x,t)$ U: const

$$\frac{\partial u}{\partial t} = U \frac{\partial u}{\partial x}$$

most accurate choice of $(\Delta x, \Delta t)$ is with $C = 1$

Explicit:

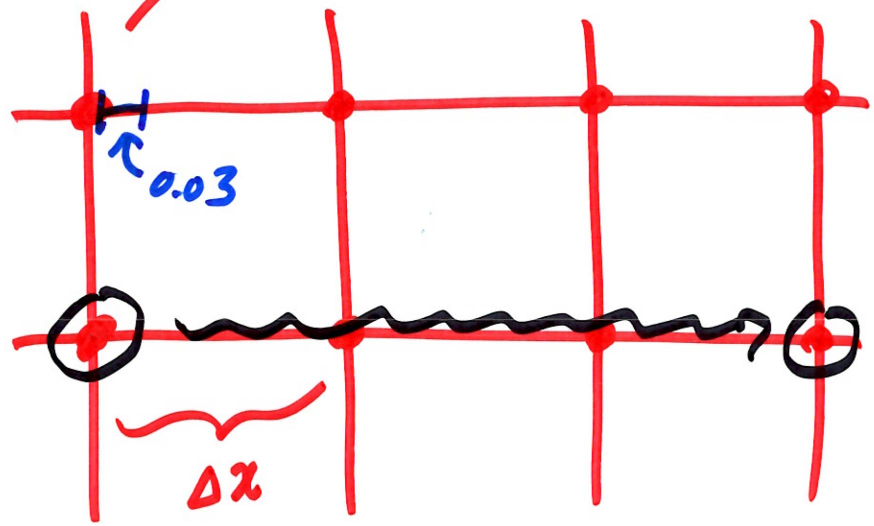


Fix Δx given U_{max}

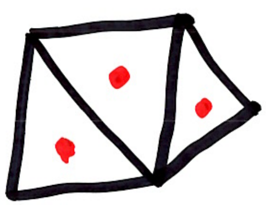
Δt

choose Δt

nodal points



FV



$U_{max} \Delta t \sim 3\ m$

Ex: fix $\Delta x = 1\ m$

choose $\Delta t = 1\ s$

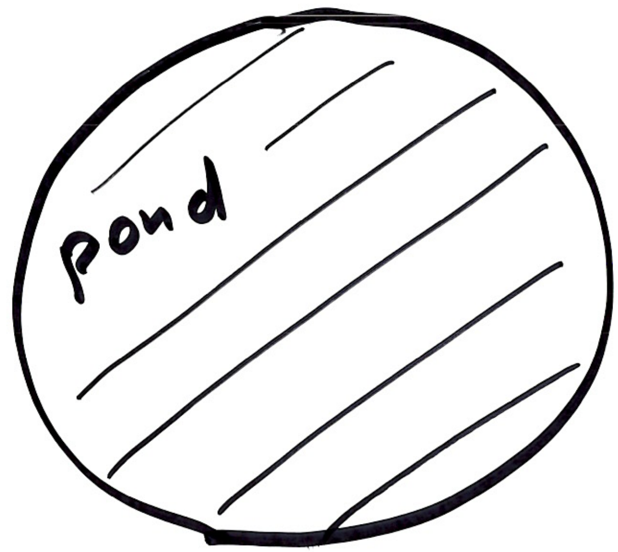
$U_{max} \sim 3\ m/s$

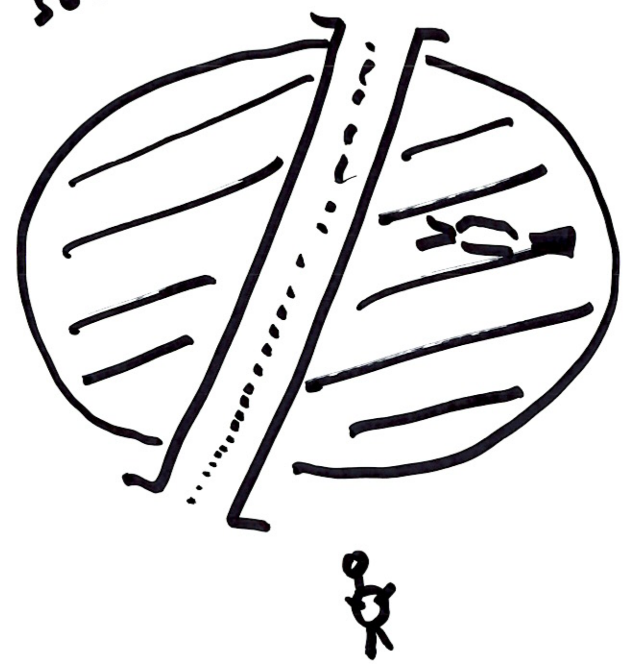
$\Delta x = 1\ m$

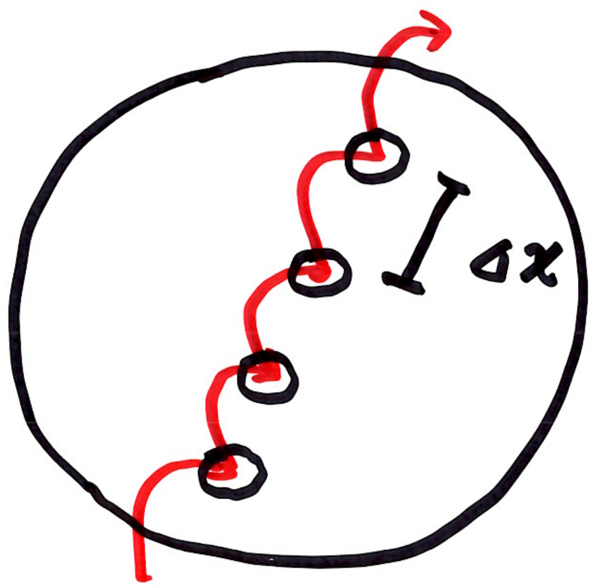
$\Delta t = 0.01\ s$

$U_{max} \approx 3\ m/s$

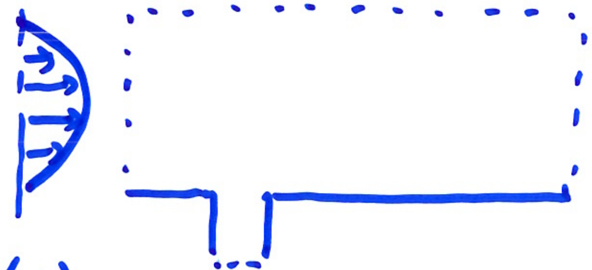
$U_{max} \cdot \Delta t \sim 0.03\ m$


pond


Continuous sol.
analytic


Δx

Proj 2 <u>Task 1b</u> ↑



$U(y)$ ↖ methane

Tip:

velocity inlet

<u>Constant</u> ↓ <u>Expression</u> ← every term must have unit of $\frac{m}{s}$

$U(y)$ velocity

$$U(y) = 0.8\,y - 0.032\,y^2$$

$y \rightarrow \underline{\underline{m}}$

non-dim

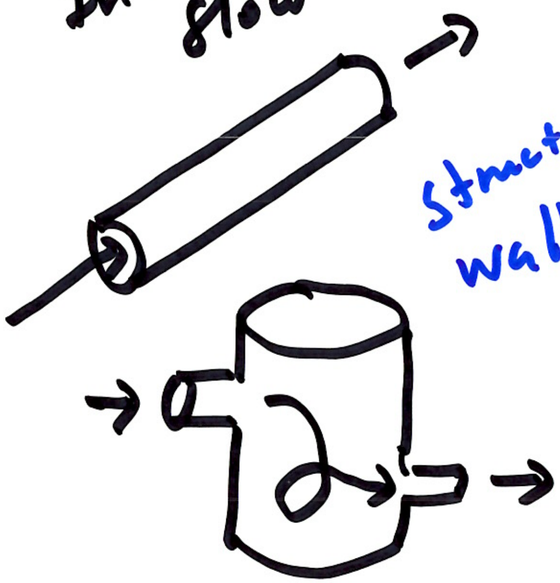
$$0.8 * \underbrace{y}_{=m} - 0.032 * \underbrace{y}_{=m} * \underbrace{y}_{=m} \quad \times$$

$$0.8[s\text{^}-1] * y - 0.032[s\text{^}-1\ m\text{^}-1] * y * y \quad \checkmark$$

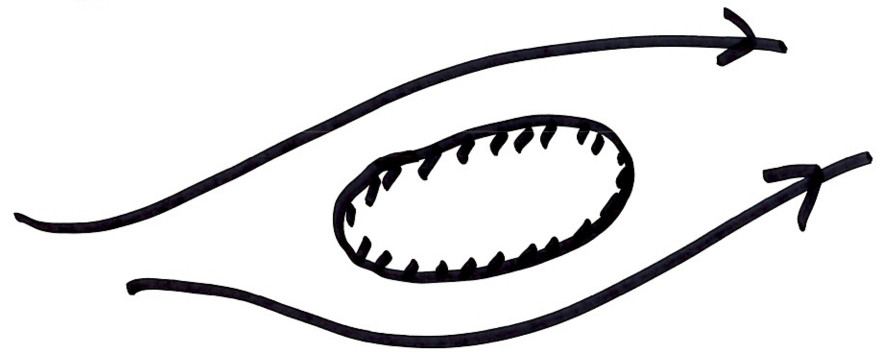
Looking ahead ... Proj 3

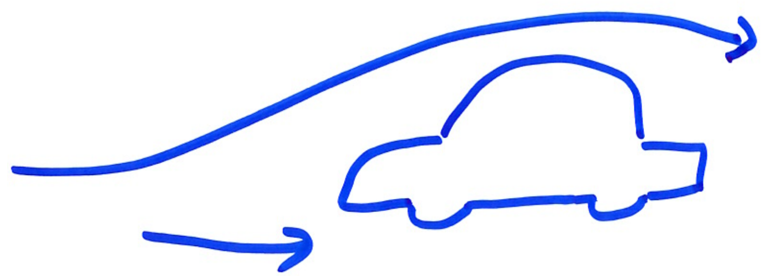
External flow

HW 1
Proj 1
Internal flow



Structure: wall



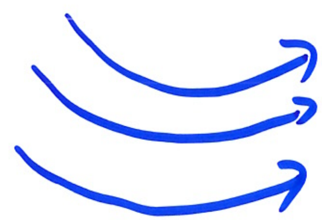


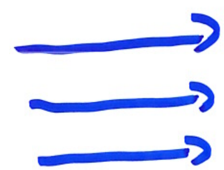
lift, drag

* Lift & drag in <u>external flow</u>

* Dependence of "flow regime" on <u>Re</u>

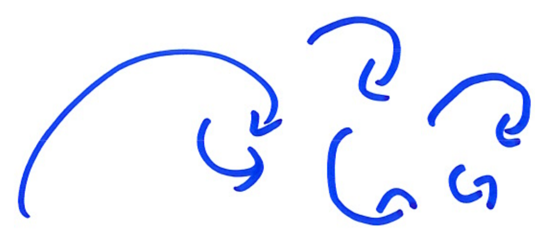
Before:





low Re → laminar

high Re → turbulence



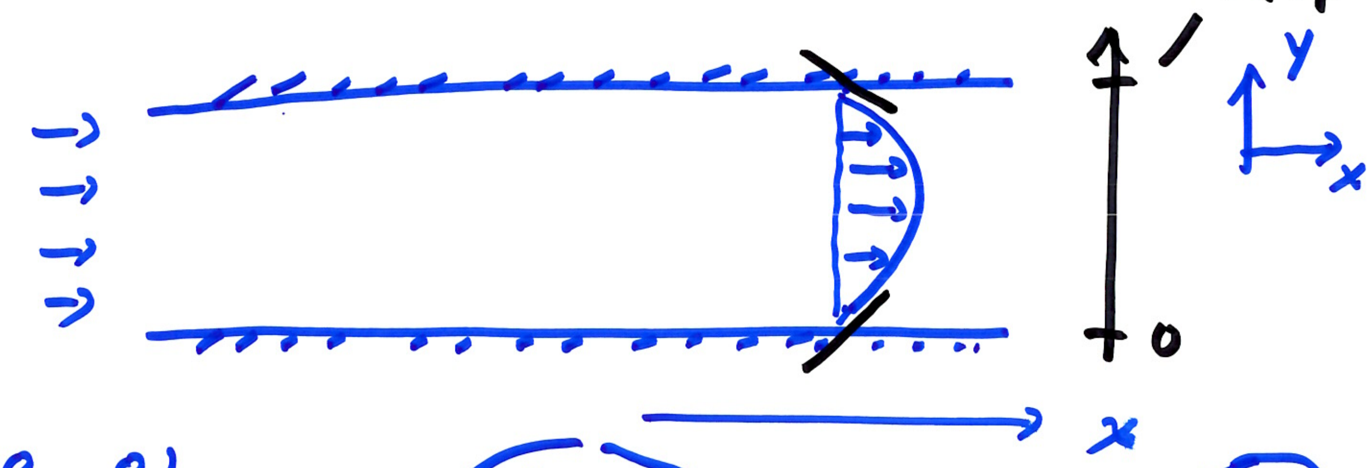
Lift/drag   force   momentum exchange

between fluid & solid structure

Internal flow

viscous term $\rho = \text{const}$

low Re



$\vec{V} \equiv (u, 0, 0)$

$\frac{\partial}{\partial x} = 0$ *

* Steady

$$\cancel{\frac{\partial u}{\partial t}} = -u \cancel{\frac{\partial u}{\partial x}} - v \cancel{\frac{\partial u}{\partial y}} - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} \right)$$

imposed

$\approx$ const $\wp$

ρu momentum $\qquad \nu = \frac{\mu}{\rho}$

$$\overset{\times}{\frac{\partial \rho u}{\partial t}} = -u\overset{\times}{\frac{\partial u}{\partial x}} - v\overset{\times}{\frac{\partial u}{\partial y}} \overset{\mathcal{P}}{\boxed{-\frac{\partial p}{\partial x}}} + \mu\left(\overset{\times}{\frac{\partial^2 u}{\partial x^2}} + \overset{\times}{\frac{\partial^2 u}{\partial y^2}}\right)$$

Total momentum $\int \rho u \, dV$

in y $\qquad \int_0^{y_{top}} (\rho u) dV \equiv M$

$$0 = \frac{dM}{dt} \equiv \frac{d}{dt}\int_{y=0}^{y=y_{top}} \rho u \, dy \equiv \int_{y=0}^{y=y_{top}} \frac{\partial(\rho u)}{\partial t} dy$$

$$= \underbrace{\int_0^{y_{top}} \mathcal{P} \, dy}_{\Pi} + \mu \int_{y=0}^{y_{top}} \frac{\partial^2 u}{\partial y^2} dy$$

$$\Pi \sim \mu \left(\underbrace{\left.\frac{\partial u}{\partial y}\right|_{y_{top}}}_{\text{Stress at wall}} - \left.\frac{\partial u}{\partial y}\right|_{y=0} \right)$$