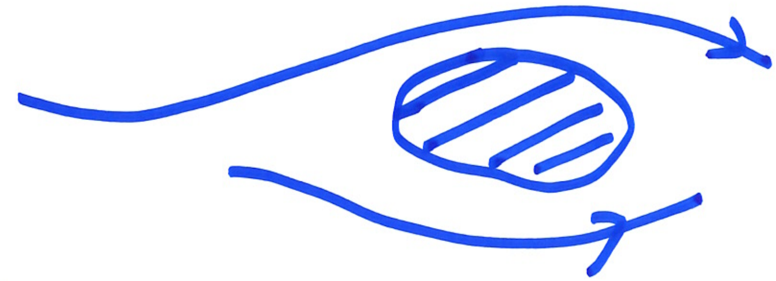
Lecture 20

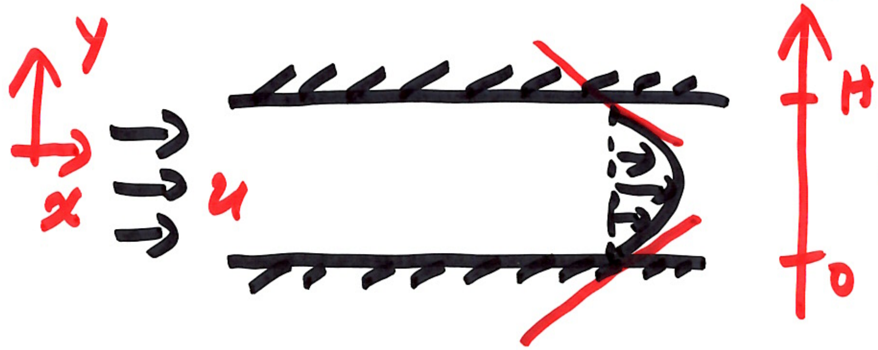
11/2

Proj 3 External flow

Recap (Lec19) ⇨



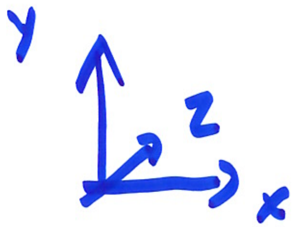
fluid &
(solid) boundary $\neq$ } lift, drag
"force"
"exchange of momentum"

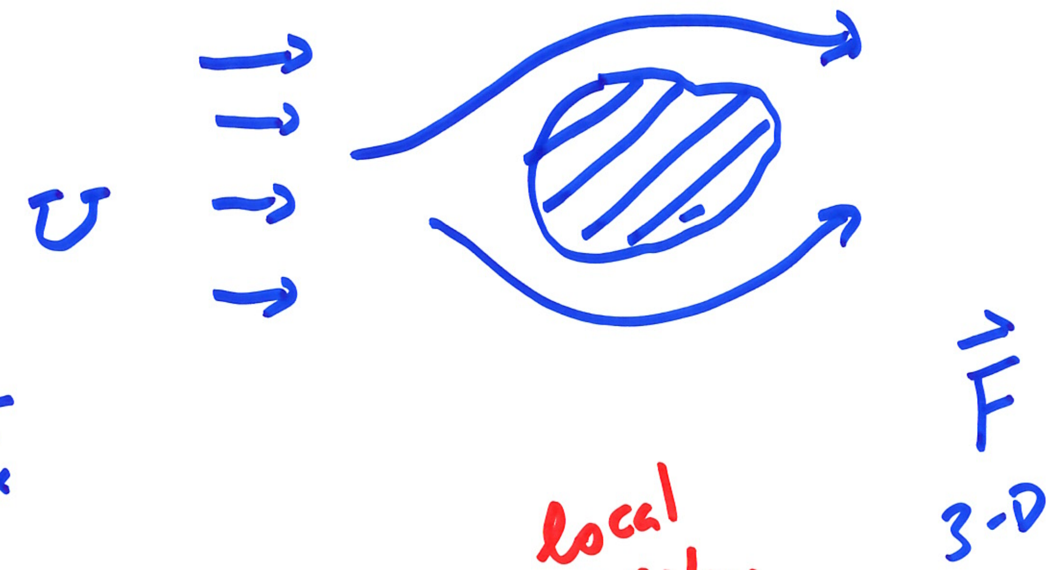


viscous effect
shear stress at boundary

$$\left.\mu \frac{\partial u}{\partial y}\right|_{\substack{\text{at } y=0 \\ y=H}}$$

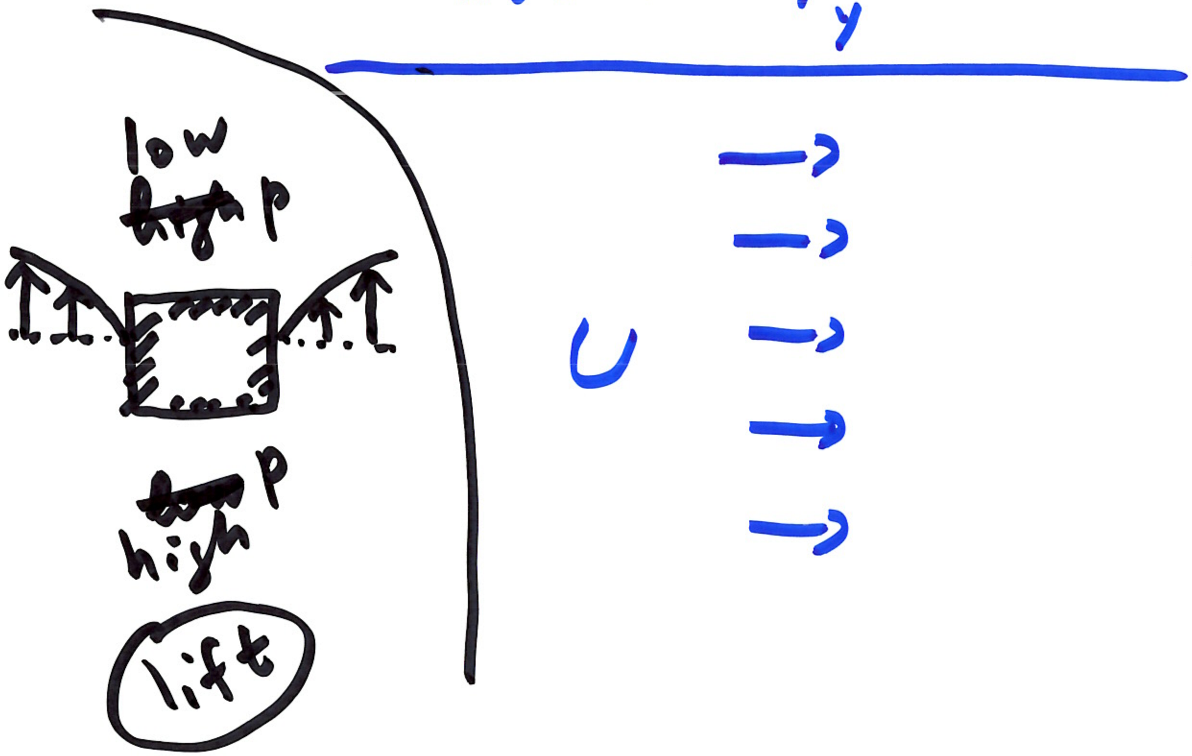
General setting (external flow)

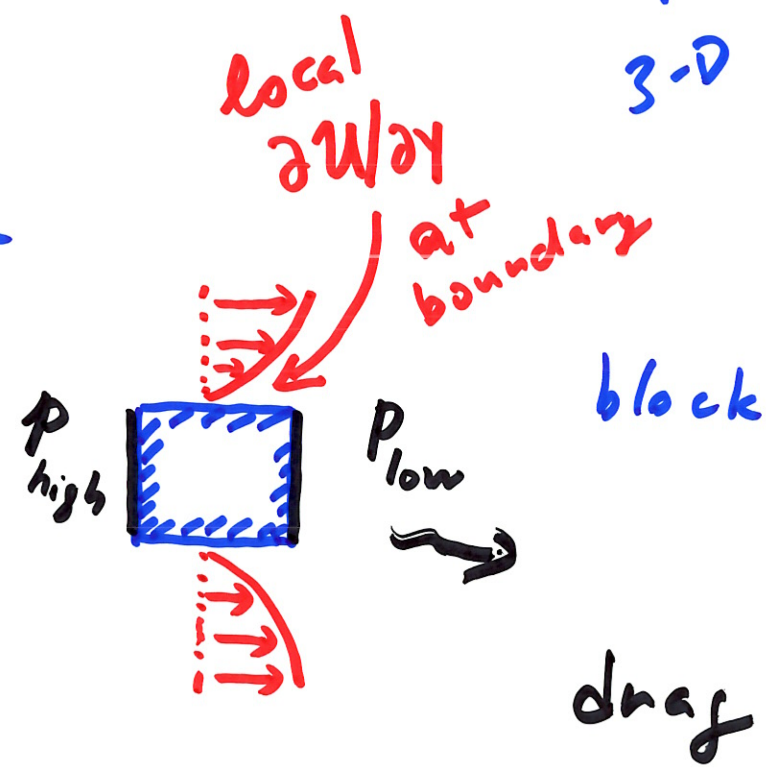




drag $\sim F_x$

lift $\sim F_y$



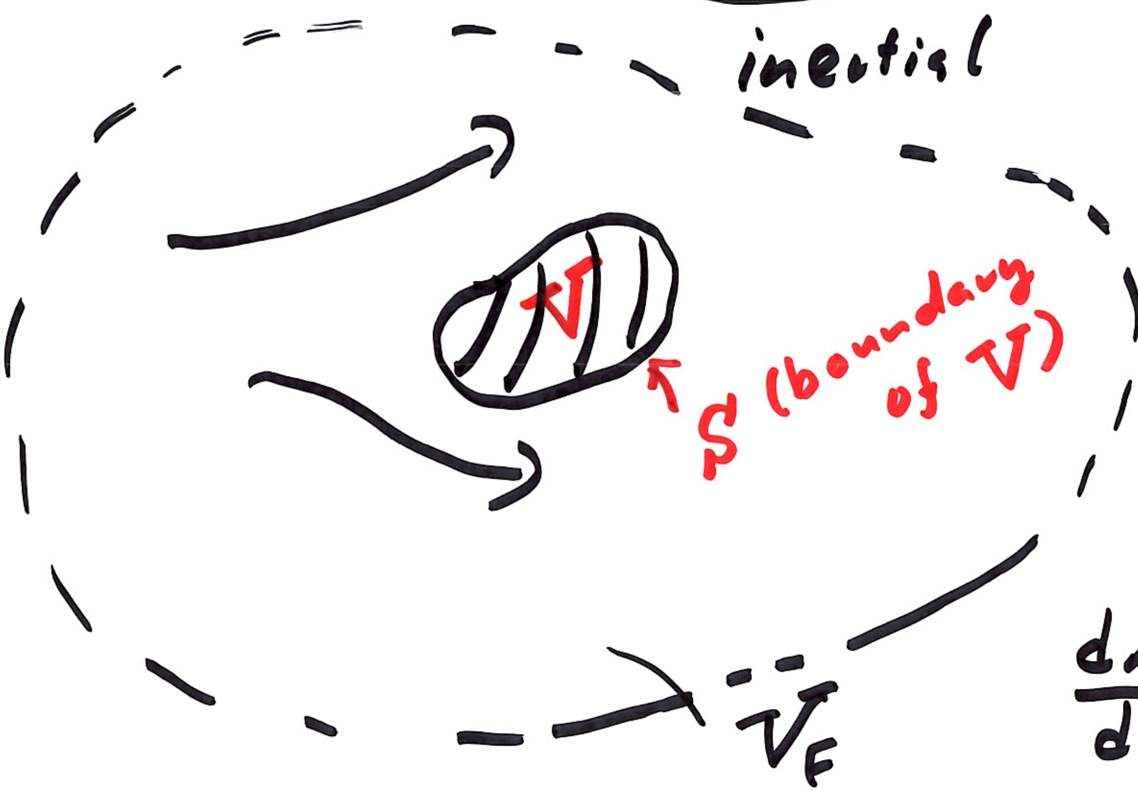


incompressible flow, const. $\underline{\underline{\rho}}$

* Consider force in $\underline{x\text{-dir}}$

momentum eq. (N-S eq) in x-dir

$$\frac{\partial u}{\partial t} = \underbrace{- \vec{V} \cdot \nabla u}_{\text{inertial}} \underbrace{- \frac{1}{\rho} \frac{\partial p}{\partial x}}_{\text{PGF}} + \underbrace{\frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)}_{\text{visc.}}$$



Total x-momentum for fluid

$$\mathcal{M}_{F,S} \equiv \iiint_{\mathcal{V}_{F,S}} \rho u \, d\mathcal{V}$$

$$\frac{d\mathcal{M}_F}{dt} = - \frac{d\mathcal{M}_S}{dt}$$

$$\Rightarrow \frac{\partial(\rho u)}{\partial t} = -\rho \vec{V}\cdot\nabla u - \frac{\partial p}{\partial x} + \mu \nabla^2 u$$

$$\frac{dM}{dt} \equiv \frac{d}{dt}\iiint \rho u \, d\mathcal{V}$$

$$\equiv \iiint \frac{\partial(\rho u)}{\partial t} d\mathcal{V}$$

$$\equiv \underbrace{\iiint -\rho \vec{V}\cdot\nabla u \, d\mathcal{V}}_{1} - \underbrace{\iiint \frac{\partial p}{\partial x} d\mathcal{V}}_{2} + \underbrace{\mu \iiint \nabla^2 u \, d\mathcal{V}}_{3}$$

Term ①

$$= -\rho \iiint \nabla\cdot(u\vec{V})\,dV \quad \text{G.D.T.}$$

$$= -\rho \oiint_S (u\vec{V})\cdot\hat{n}\,dS$$

$$= 0$$

b.c. for $\vec{V}$:
no slip
($\vec{V} = 0$ at boundary)

inertial term does not contribute to lift/drag

* incompressible.

~~$\rho = 0$~~

$\rho = \text{const}$

Continuity eq.

$$\underline{\nabla\cdot\vec{V} = 0}$$

$$\Rightarrow \quad \vec{V}\cdot\nabla u = \nabla\cdot(u\vec{V})$$

$$\equiv \cancel{u\,\nabla\cdot\vec{V}} + \vec{V}\cdot\nabla u$$

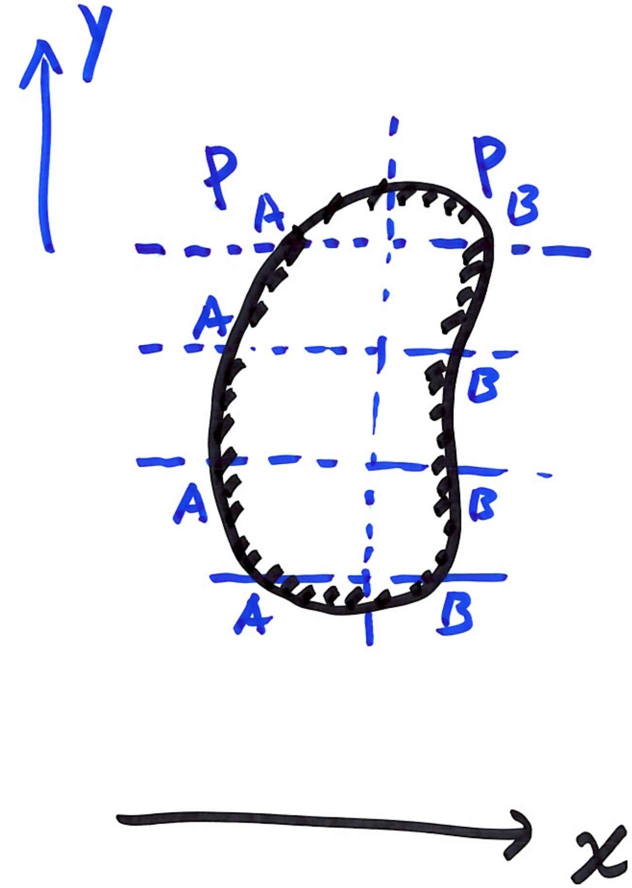
Term 2

$$= -\iint \left[\int_A^B \frac{\partial p}{\partial x}\, dx \right] dy\, dz$$

$$= -\iint \left[p(B) - p(A) \right] dy\, dz$$

$$= \iint \left[\underline{p(A)} - \underline{p(B)} \right] dy\, dz$$

"pressure ~~term~~" term



Term ③

$$= \mu \iiint \nabla \cdot (\nabla u)\, dV$$

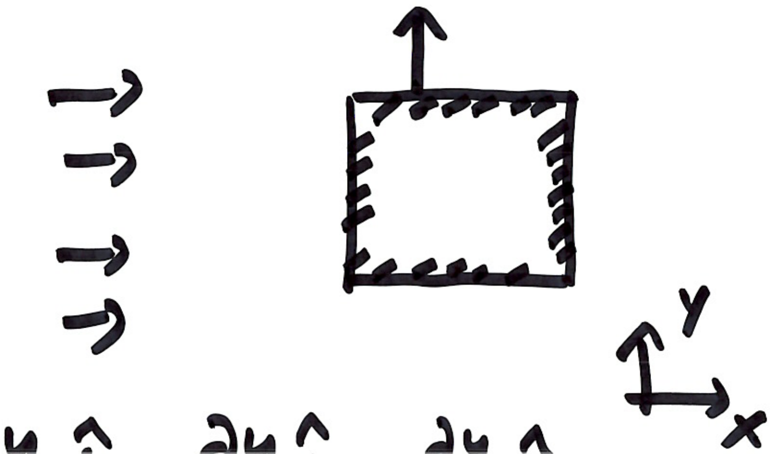
$$= \underline{\underline{\mu}} \oint\!\!\oint_S (\nabla u) \cdot \hat{n}\, dS$$

"shear stress" or "viscous" term

$$\nabla^2 u \equiv \nabla \cdot (\nabla u)$$

G.D.T.

$\hat{n} = (0, 1)$



$$\nabla u = \frac{\partial u}{\partial x}\hat{i} + \frac{\partial u}{\partial y}\hat{j} + \frac{\partial u}{\partial z}\hat{k}$$

$$(\nabla u) \cdot \hat{n} = \frac{\partial u}{\partial y}$$

$$\mu \frac{\partial u}{\partial y}$$

pressure term $\longleftarrow$ P.G.F.

shear stress/viscous $\longleftarrow$ visc.

$\nu \equiv \frac{\mu}{\rho}$

$$\frac{\partial \vec{V}}{\partial t} = \underbrace{- \vec{V} \cdot \nabla \vec{V}}_{\text{I}} \underbrace{- \frac{1}{\rho} \nabla p}_{\text{P}} + \underbrace{\nu \nabla^2 \vec{V}}_{\text{V}}$$

$$Re \sim \frac{|\text{I}|}{|\text{V}|} \sim \frac{\frac{U^2}{L}}{\nu \frac{U}{L^2}} \sim \frac{UL}{\nu}$$

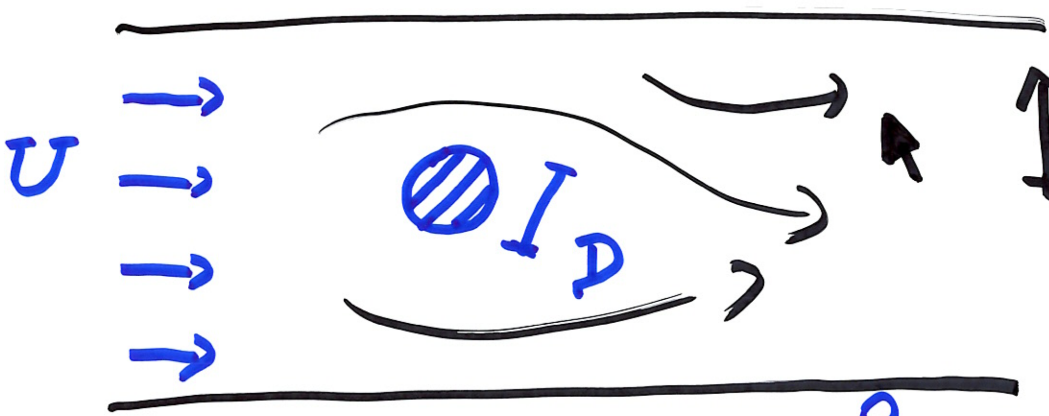
- high Re (turbulent) $\quad \text{I} \gg \text{V}$
- low Re (laminar) $\quad \text{V} \gg \text{I}$

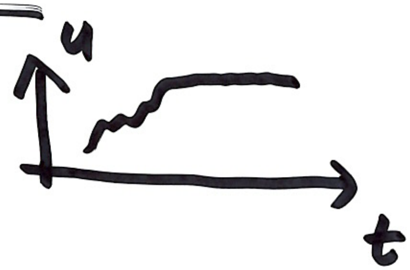
primary balance

$\text{P} \sim \text{I}$

$\text{P} \sim \text{V}$

"Flow regimes" of external flow

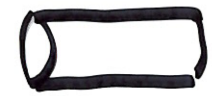




$Re \sim \frac{U \cdot D}{\nu}$

flow behavior in terms of

1. spatial structure
2. temporal behavior



low Re

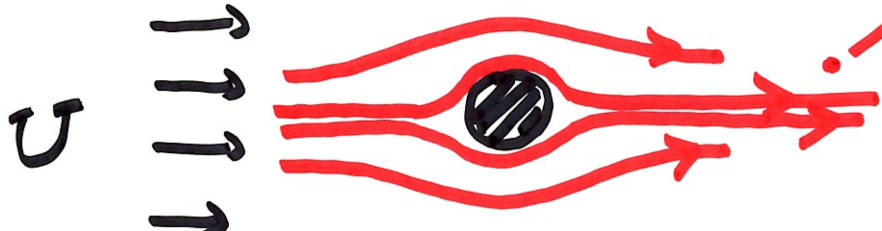
laminar

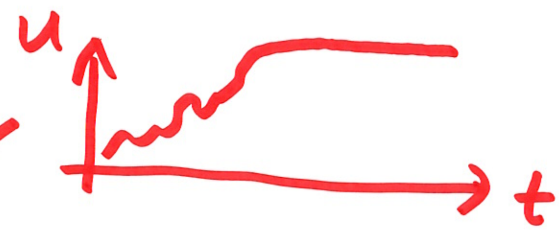
high Re

turbulent

NOT SO SIMPLE!

<u>Very low Re</u> $Re \sim 1$



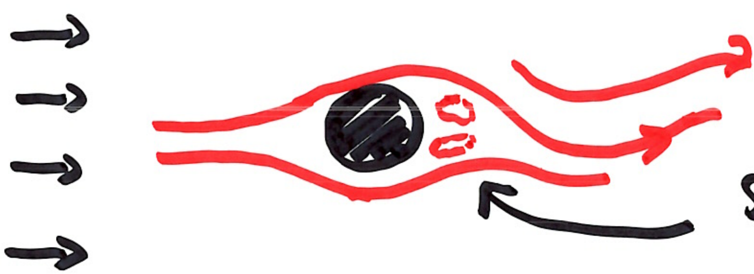


steady in t

simple, symmetric streamline

moderate Re 10^2

(wake)

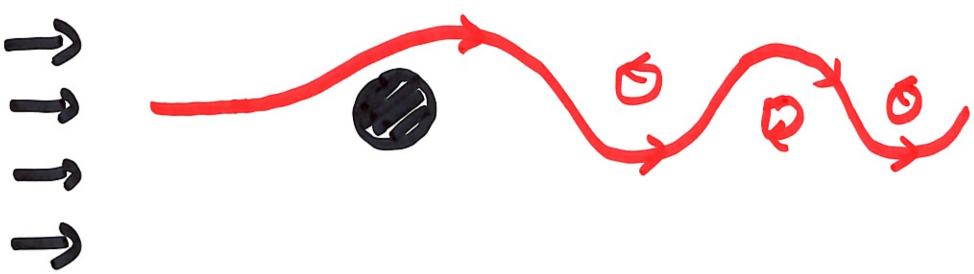


structures downstream of

oscillatory in t

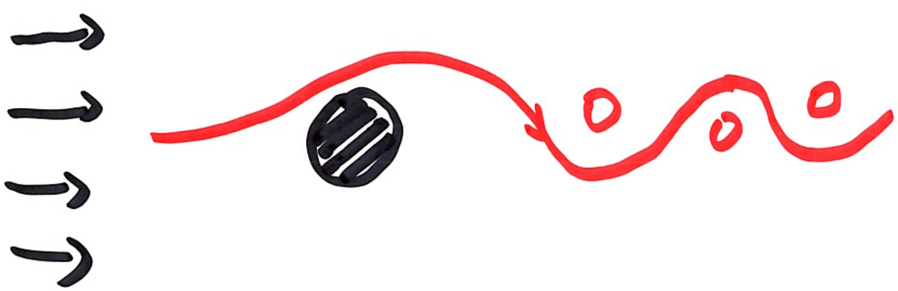
further increase <u>Re</u> $10^2 - 10^3$

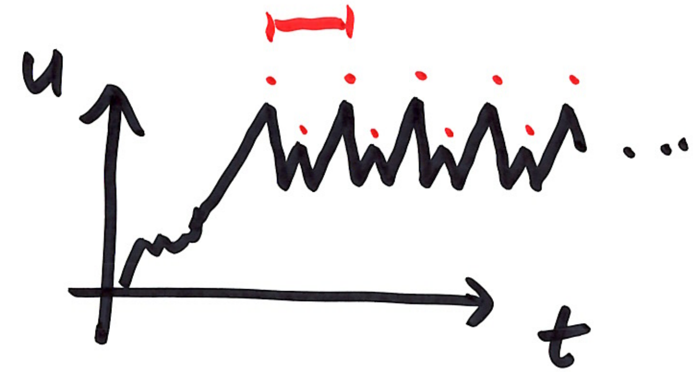
Karman "vortex street"



* lift & drag also oscillate in time

Further increase Re





multiple-periodic
quasi-periodic

further increase Re $\sim 10^5$

chaotic
"turbulent wake"

chaotic