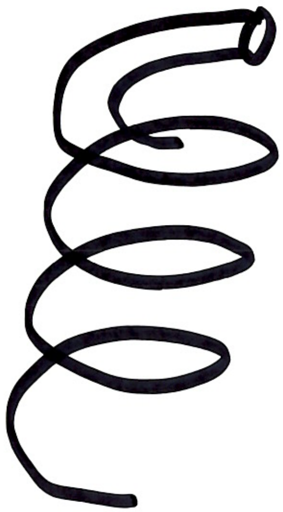
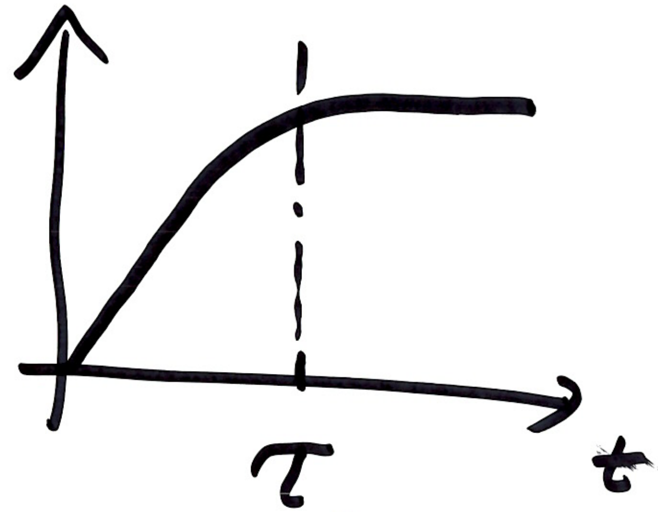
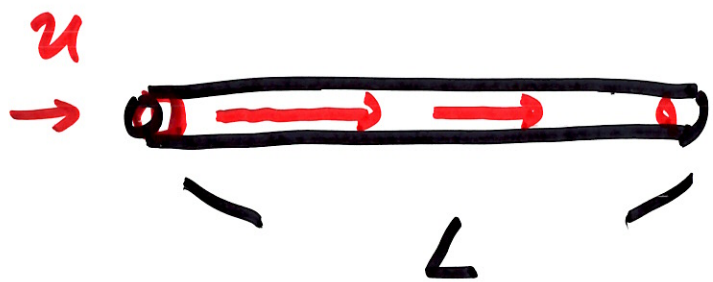
Lecture 21

11/4



Task 2b





$\tau \approx L/u$

~~$L\tau = u$~~

$u\tau = L$

Task 4 a, b.

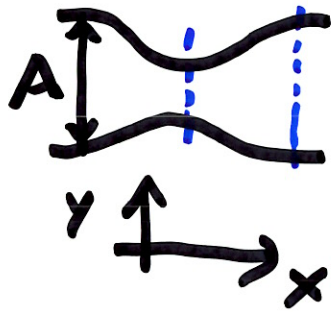
(b) ρ = const

consider "1-D" case — more idealized than Task 4

steady

$u(x)$

$A(x)$ etc.



mass continuity

$$\rho u A = \text{const} \Rightarrow uA = \text{const}$$

$$d(uA) = 0 \Leftarrow u \propto \frac{1}{A}$$

$$\boxed{\frac{du}{u} = -\frac{dA}{A}}$$

(a) $\rho \neq \text{Const}$

$p = \rho R T$

$\rho = \frac{p}{RT}$

Need thermodynamics

1st law of

$0 = dU + pdV$ $\propto c_v dT$

or $= dH - Vdp$

eq. of mass

$d(\rho u A) = 0$

$\Rightarrow \frac{du}{u} + \frac{d\rho}{\rho} = -\frac{dA}{A}$ — (1)

if small Proj^n Task^n

adiabatic

incompressible

$\rho = 0$

$0 = dU + pdV$

decoupled

Compare: (b)

$\frac{du}{u} = -\frac{dA}{A}$

eq. of momentum (inviscid)

$$\cancel{\frac{\partial \vec{v}}{\partial t}} = -\vec{v}\cdot\nabla\vec{v} - \frac{1}{\rho}\nabla p$$

N.S. eq.

$\nu = 0$

1-D steady

$$0 = -u\,du - \frac{1}{\rho}\,dp \qquad \text{—— (2)}$$

(2)

$$0 = -u\frac{du}{dx} - \frac{1}{\rho}\frac{dp}{dx}$$

eq of energy (1st law) — adiabatic

$$0 = c_p\,dT - \alpha\,dp \qquad \alpha = \frac{1}{\rho}$$

$$= c_p\,dT - \frac{1}{\rho}\,dp \qquad \text{—— (3)}$$

eq. of state $p = \rho R T$

$$\rho = \frac{p}{RT}$$

$$\frac{d\rho}{\rho} = \frac{dp}{p} - \frac{dT}{T} \quad \text{---- (4)}$$

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(1), (2), (3), (4) :

$$\left(1 - \frac{u^2}{\gamma RT}\right) \frac{du}{u} = -\frac{dA}{A}$$

$$\gamma = C_p / C_v$$

speed of sound

$$c = \sqrt{\gamma RT}$$

Mach # $M \equiv \frac{u}{c}$

Compressible Task 4a

$$(1-M^2)\frac{du}{u} = -\frac{dA}{A}$$

If $M \ll 1$

If M never exceeds 1

$\Rightarrow (1-M^2) > 0$

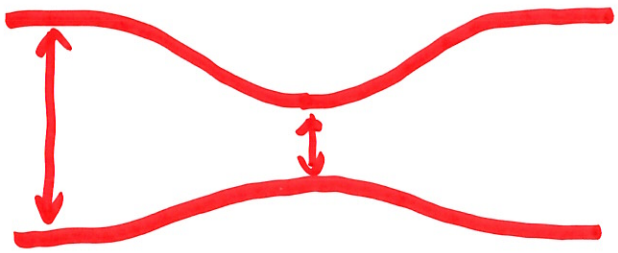
If $M > 1$ at neck

$(1-M^2) < 0$

Task 4b

$\rho = \text{const}$

$$\frac{du}{u} = -\frac{dA}{A}$$



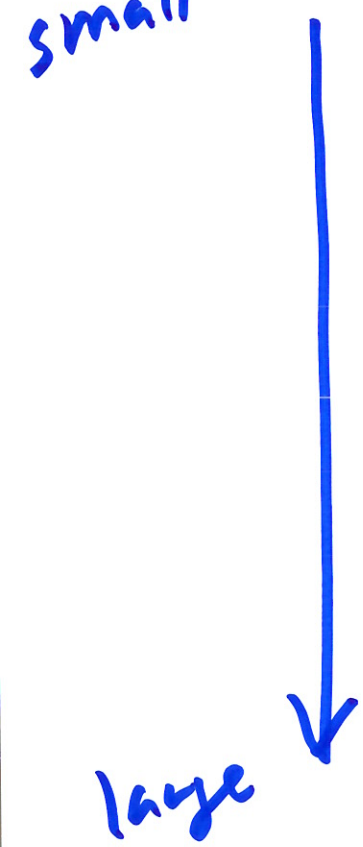
Recap (Lec 20)

$$Re \sim \frac{UD}{\nu}$$

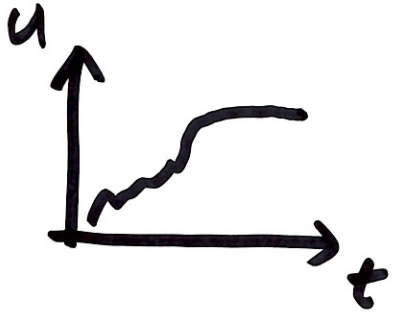
Re U D

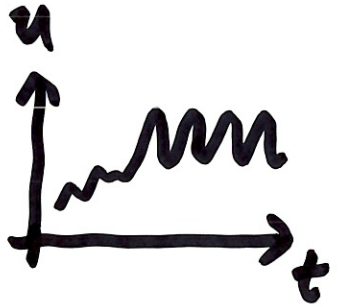
small

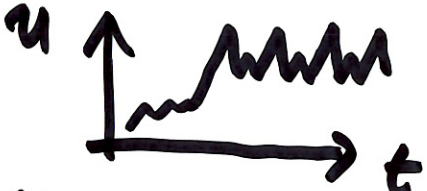
large



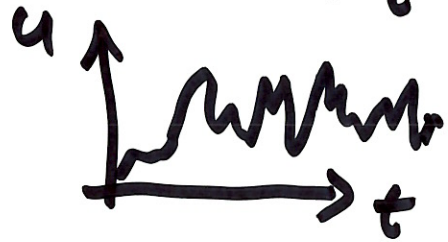
steady

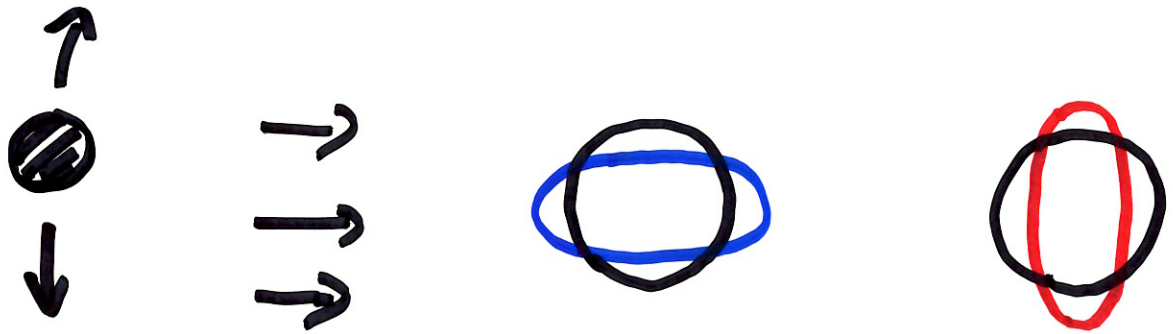






turbulent





CFD

fix the cylinder

lift F_L on the cylinder