

## Relationship Between Linear and Angular Motion

A very important feature of human motion...

Segment rotations combine to produce linear motion of the whole body or of a specific point on a body segment or implement (e.g., a distal location on the segment or implement)

e.g., running...

coordinated joint rotations result in translation of the entire body

e.g., softball (underhand) pitch...

goal is to generate high linear velocity of the ball at release

e.g., hitting a golf ball...

goal is to maximize clubhead speed at impact

Key concept: The motion of any point on a rotating body (e.g., bicycle wheel) can be described in linear terms (i.e., curvilinear motion).

Key information: axis of rotation, location of point of interest relative to axis.

## Mathematical Relationships between Linear and Angular Motion

### 1. Combining linear and angular distance:

$$\ell = \phi \cdot r$$

Linear distance that point of interest is located from the axis of rotation: i.e., "radius of rotation"

Linear distance that the point of interest travels

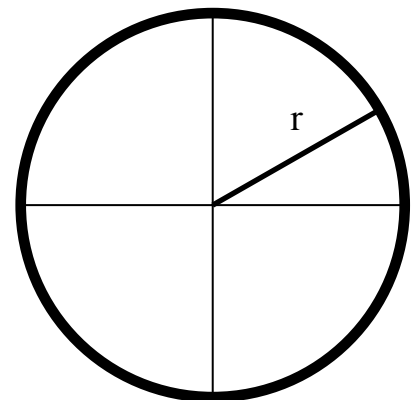
Angular distance through which the rotating object travels

**\*\*WARNING\*\***

$\phi$  must be expressed in \_\_\_\_\_  
for this expression to be valid!

e.g., bicycling electronic odometer

Device effectively measures distance \_\_\_\_\_ per wheel rotation for a point on the outer edge of the tire...



You set  $r$  (bicycle wheel radius) e.g., 33 cm  
 Device counts rotations (1 rev =  $2\pi$  rad)

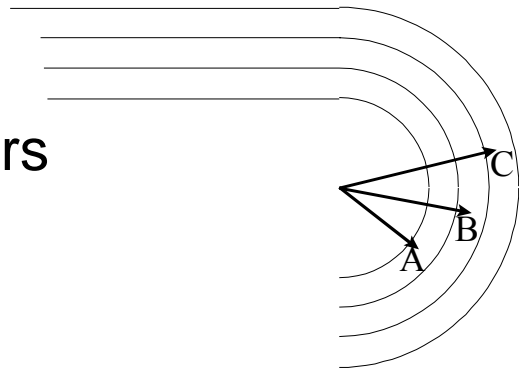
$$\ell = \phi \cdot r = (6.28 \text{ rad})(33 \text{ cm}) = 207.4 \text{ cm}$$

i.e., bike moves 2.074 m per wheel rotation

e.g., Staggered start in 200 m race in track

Object: to equate the linear distance traveled by all sprinters through the curve

$$\ell = \phi \cdot r$$



The farther a sprinter is positioned from the center of the turn (i.e., as  $r$  increases), the smaller the angular distance that must be traveled to cover a given linear distance.

What of the lanes does a sprinter prefer?  
 Mechanically?  
 Psychologically?

Question: How much farther would you run when completing 4 laps on a 400 meter track if you chose to use lane 8 instead of lane 1?

Any difference in distance traveled in the straightaways?

What do you need to know?

## 2. Combining linear and angular velocity:

$$v_T = \omega r$$

Radius of rotation of point of interest

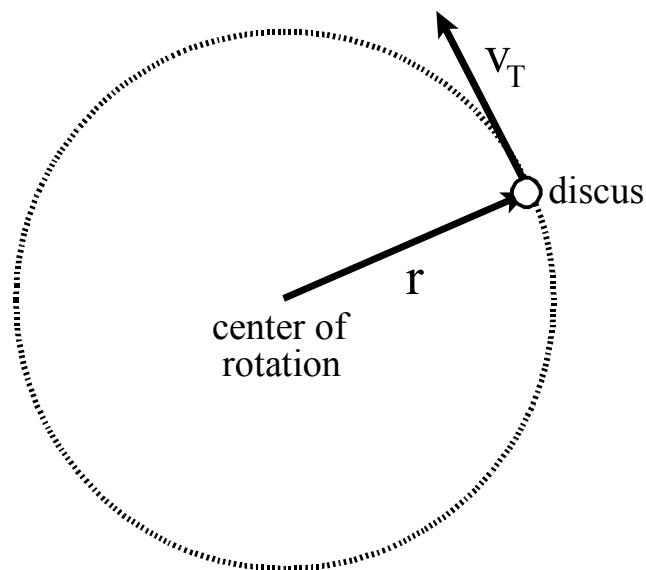
Tangential velocity (i.e., linear velocity - at any instant, it is oriented along a tangent to the curved path

Angular velocity of the rotating body (*must be expressed in \_\_\_\_\_*)

e.g., golf - clubhead velocity

e.g., baseball - linear velocity of bat at point of contact with ball

e.g., Discus throw - a critical performance factor:  $v_T$  of discus at instant of release



A discus thrower is rotating with a velocity of 1180 deg/s at the instant of release. If the discus is located 1.1 m from the axis of rotation, what is the linear speed of the discus?

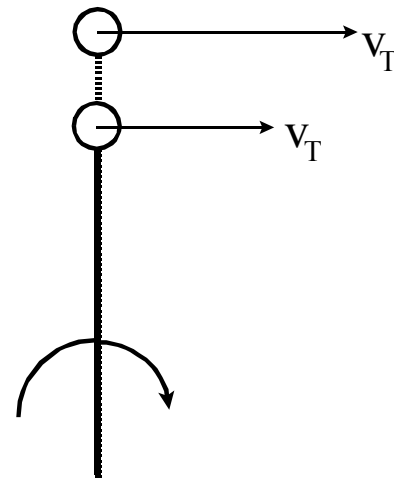
$$\omega = (1180 \text{ deg/s}) (1 \text{ rad}/57.3 \text{ deg}) = \underline{\hspace{2cm}}$$

$$v_T = \omega r = (20.6 \text{ rad/s})(1.1 \text{ m}) = \underline{\hspace{2cm}}$$

e.g., Recall our discussion of shoulder internal rotation in throwing...

If shoulder internal rotation was the only motion used to generate ball velocity and the shoulder's angular velocity at ball release was 1800 deg/s, how fast would the ball be traveling linearly at release?

What do you need to know?



If two individuals had the same internal rotation velocity but one had a longer arm than the other, which would generate the higher ball velocity?

What does  $v_T = \omega r$  tell us about performance?

In many tasks, an important biomechanical goal is to \_\_\_\_\_  $v_T$  (i.e., the linear velocity of a point located distally on a rotating body)...

e.g., golf clubhead velocity, the ball in throwing for maximum speed or distance

Theoretically,  $v_T$  can be increased if...

$\omega$  can be increased while  $r$  is maintained, or  $r$  can be increased while  $\omega$  is maintained.

Problem: It is difficult for an athlete to maintain  $\omega$  if  $r$  is increased and vice versa.

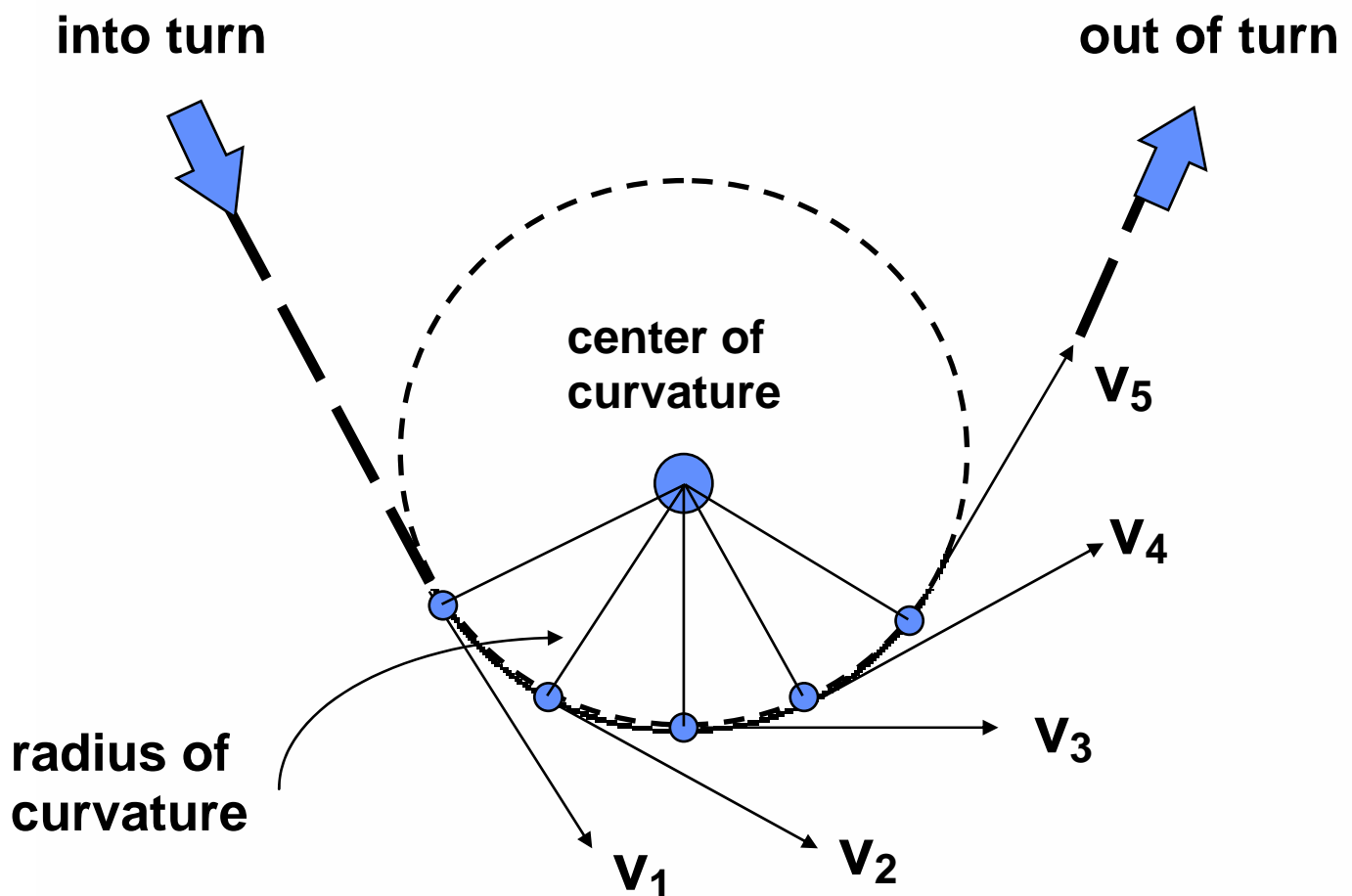
e.g., it is harder to maintain an object's rotation when its mass is distributed farther from the axis.

(A full explanation of this will have to wait until we talk about angular \_\_\_\_\_.)

### 3. Combining linear and angular acceleration:

Radial acceleration ( $a_R$ ) - the linear acceleration that serves to describe the rate of change in direction of an object following a curved path.

e.g., while running around a curved path at constant speed:



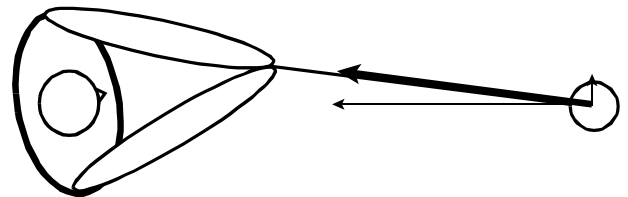
Note:  $v_1=v_2=v_3=v_4=v_5$  which means that \_\_\_\_\_ is not changing.

However, \_\_\_\_\_ **is** changing. WHY?



In order for the runner to accomplish this continual change in direction (i.e.,  $a_R$ ), there must be a **FORCE** on the runner pushing toward the center of the curvature. This is called centrifugal force. The runner gets this force from pushing laterally outward against the ground. The ground, in turn, pushes inward against the runner (toward the center of curvature). (More on this later when we discuss kinetics.)

Another example: As a hammer thrower spins just before release of the hammer, the hammer (ball on the end of the cable) follows a curved path because of the restraining effect of the cable.



Equation for radial acceleration:

$$a_R = \frac{v^2}{r} = \omega^2 r$$

This demonstrates that:

- A. for a given  $r$ , higher  $v_T$  is related to a higher  $a_R$ ; which means a higher force is needed to produce  $a_R$  (i.e., to maintain the curved path).
- B. for a given  $v_T$ , lower  $r$  (i.e., tighter "turning radius") results in a higher  $a_R$  (and the need for a greater force to maintain curved path).

e.g.,  $a_R$  of an Indy car driver through a curve?

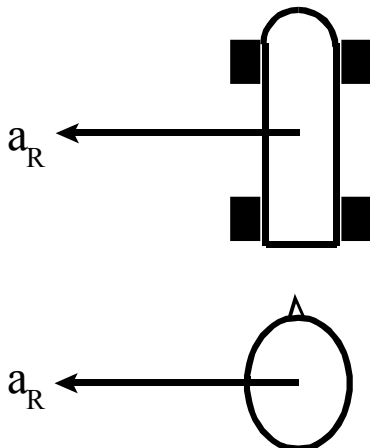
$v_T$  = car speed e.g., 200 mph = 89.4 m/s

$r$  = radius of turn e.g., 1/4 mi = 403 m

$$a_R = v_T^2 / r$$

$$a_R = (89.4 \text{ m/s})^2 / (403 \text{ m}) = \underline{\hspace{2cm}}$$

i.e., about 2g's



Consider inertia of head: need for strap to assist neck muscles in maintaining upright head position.

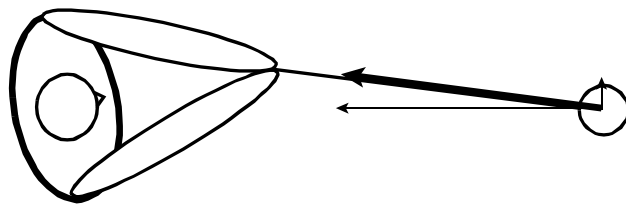
Tangential acceleration ( $a_T$ ) - the linear acceleration that serves to describe the rate of change in magnitude of tangential velocity (i.e., the rate at which one is speeding up or slowing down).

Equations for tangential acceleration:

$$(1) a_T = \quad =$$

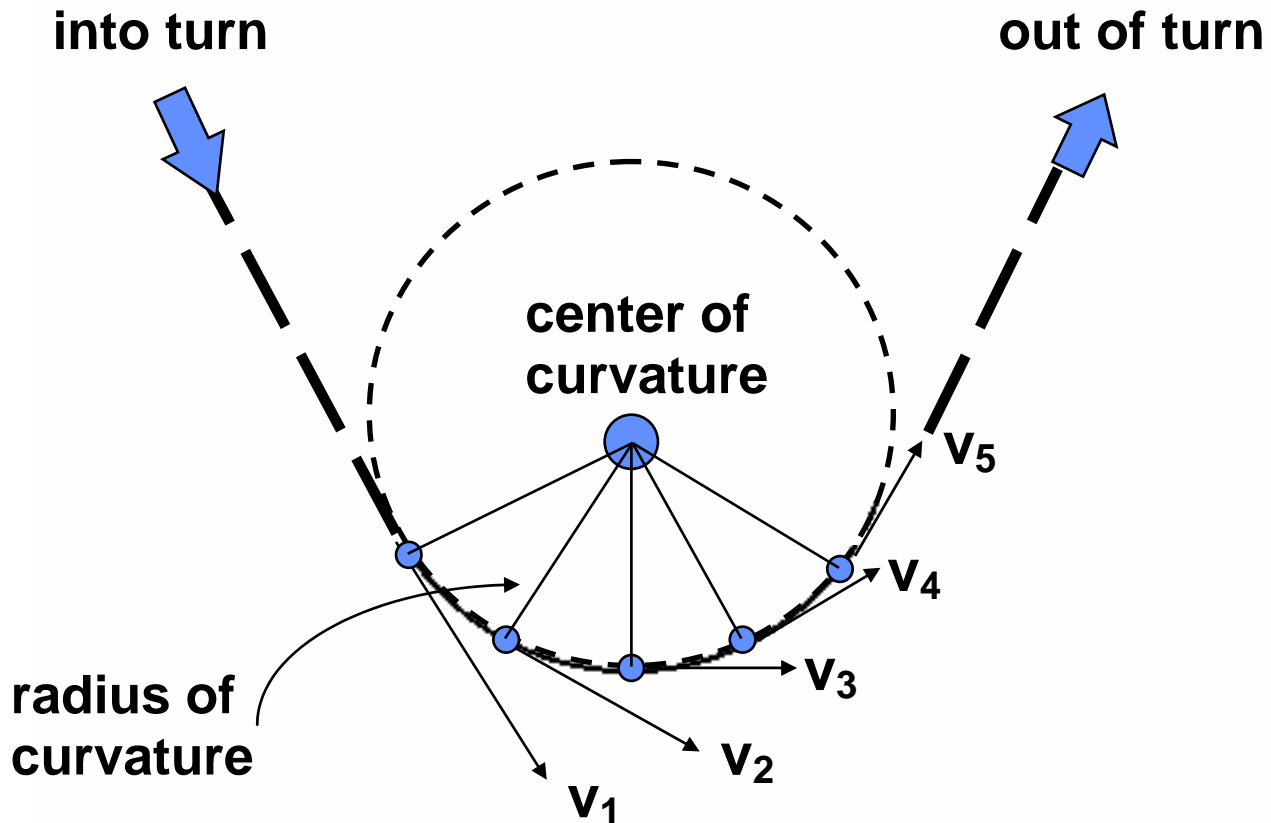
$$(2) a_T = \quad \text{(Watch your units! Why?)}$$

e.g., goal of a hammer thrower's prerelease rotations is to increase the hammer's linear speed.



What forces are applied to a hammer during the body rotations prior to release?

Another example: Running around a curved path at a non-constant speed (usually while slowing down):



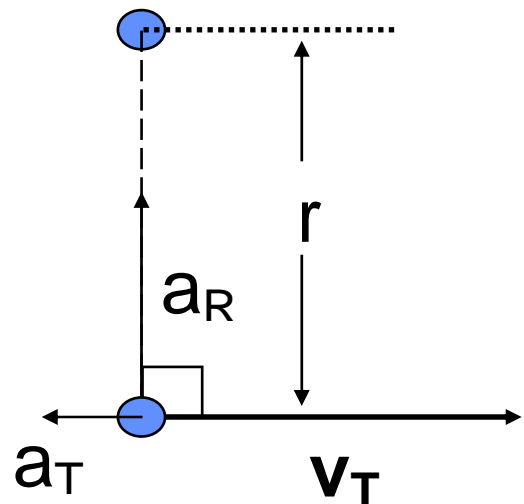
Speed is decreasing ( $v_1 > v_2 > v_3 > v_4 > v_5$ ) Velocity is both decreasing in magnitude and changing in direction.

In the above example (running around a curve while slowing down) say  $v_2 = 10$  m/s,  $v_3 = 8$  m/s,  $v_4 = 6$  m/s  $t_2 = 39.2$  s,  $t_3 = 39.5$  s,  $t_4 = 39.8$  s.

What is the average tangential acceleration during the time interval  $t_2$  to  $t_4$  (in both  $\text{m/s}^2$  and  $g$ 's)?

What is the runner's radial acceleration (in both  $\text{m/s}^2$  and  $g$ 's)?

Total acceleration (resultant acceleration) is the vector sum of radial and tangential accelerations at any given point in time.



The resultant force also points in the same direction as the resultant acceleration, so if you can figure the way the acceleration points, you can figure out which way the force points. (Oops, that is kinetics! We'll have to wait until later to expand on this.)