Hollow vs. Solid Bone Example

KIN 412/512

Problem to illustrate strength of tubular materials vs solid materials

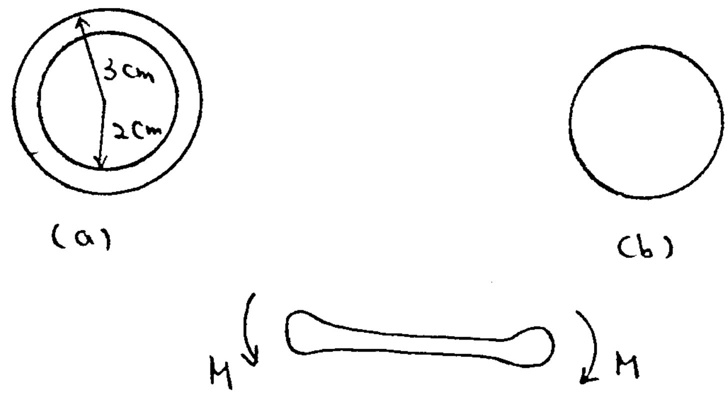


Figure: P1

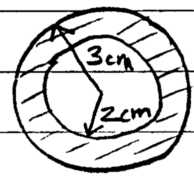
BioE 505
Mid-Term Exam
Open Book, 75 minutes

1. The approximate cross-section geometry in the mid-section of a long bone is shown in Figure P-1(a). If a solid bone does not contain a marrow cavity but has the same cross-sectional area [Fig. P-1(b)], and the two bones are subjected to the same bending moment M, compare their bending strength. If we would like to increase the bending strength of the fictitious bone of Figure P-1(b), we can increase the diameter. How much would the total bone weight increase if the bending strength of the solid bone is increased to that of the bone shown in Figure P-1(a). (20 points)

Rick Hinrichs
BioE 505
Midterm Exam
1-9-80

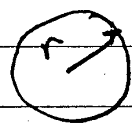
good

#1.

a) 

$r_1 = 2$
$r_2 = 3$

$\pi(3^2 - 2^2) = 5\pi$ cross sectional area hollow bone

b) 

$\pi r^2 =$ cross sectional area of solid bone

for same cross sectional area

$\pi r^2 = 5\pi \Rightarrow \boxed{r = \sqrt{5}}$ ✓

[$\sigma_0 = \frac{Mc}{I}$ max bending stress in beam]

for hollow bone:

$\sigma_{oh} = \frac{M r_2}{I_h}$ where $I_h = \frac{\pi}{4}(r_2^4 - r_1^4)$

$= \frac{\pi}{4}(3^4 - 2^4)$

$= \frac{M(3)}{16.25\pi}$ $\qquad = \frac{65\pi}{4} = 16.25\pi$

$\sigma_{oh} = .058764902\ M$

for solid bone

$\sigma_{os} = \frac{Mr}{I_s}$ where $I_s = \frac{\pi}{4} r^4 = \frac{25\pi}{4} = 6.25\pi$

$= \frac{M\sqrt{5}}{6.25\pi}$

$\sigma_{os} = .113882\ M$ ✓

(2)

1 cont'd

Thus $\sigma_{oh} < \sigma_{os}$

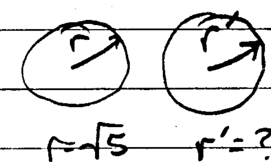
$(.059M < .114M)$

∴ a given moment M produces less of a maximum bending stress in the hollow bone

If solid bone increased in diameter so that $\sigma_{oh} = \sigma_{os}$ – ~~now max~~ what should this diameter be?

$$\sigma_{os} = \frac{Mr'}{I_s} = \frac{Mr'}{\frac{\pi}{4}r'^4} = \frac{4M}{\pi r'^3}$$

set = to σ_{oh} $(= .059M)$



$$\frac{4M}{\pi r'^3} = .059M$$

$$r' = \sqrt[3]{\frac{4}{.059\pi}}$$

$$r' = 2.7878155 \text{ cm}$$

Weight is proportional to cross sectional area (πr^2)

% Increase in weight: $\frac{r'^2}{r^2} = \frac{7.77}{5} = 1.554383$ ✓

∴ 55.4% increase in weight for solid bone to have same max bending stress as hollow bone for a given moment