Law of Cosines and Law of Sines
(applicable to any triangle)

Law of cosines:
\[ c^2 = a^2 + b^2 - 2ab \cos \gamma \]
\[ b^2 = c^2 + a^2 - 2ac \cos \beta \]
\[ a^2 = b^2 + c^2 - 2bc \cos \alpha \]

[Note: for a 90° angle
Law of cosines turns into the Pythagorean Theorem since \( \cos 90° = 0.000 \)]

Law of sines:
\[ \frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} \]
Vector Addition using Law of Cosine's and Law of Sines

**Example**
Find the magnitude and direction of the resultant of the two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ shown in Figure 1 if $|\mathbf{F}_1| = 87.2$ pounds and $|\mathbf{F}_2| = 53.8$ pounds.

**Solution**
We complete the parallelogram, as shown in Figure 2. The angle $\theta$ is the supplement of $39.7^\circ$:

$$\theta = 180^\circ - 39.7^\circ = 140.3^\circ$$

![Figure 1](image1)

![Figure 2](image2)

By the law of cosines,

$$|\mathbf{R}|^2 = |\mathbf{F}_1|^2 + |\mathbf{F}_2|^2 - 2|\mathbf{F}_1||\mathbf{F}_2|\cos \theta$$

$$= (87.2)^2 + (53.8)^2 - 2(87.2)(53.8)\cos 140.3^\circ$$

$$= 17,717.34$$

so $|\mathbf{R}| = 133.1$ pounds. We indicate the direction of $\mathbf{R}$ by finding the angle $\alpha$ by the law of sines.

$$\frac{|\mathbf{F}_2|}{\sin \alpha} = \frac{|\mathbf{R}|}{\sin \theta}$$

$$\sin \alpha = \frac{|\mathbf{F}_2|\sin \theta}{|\mathbf{R}|} = \frac{(53.8)\sin 140.3^\circ}{133.1} = 0.2582$$

Thus, $\alpha = 15.0^\circ$. 