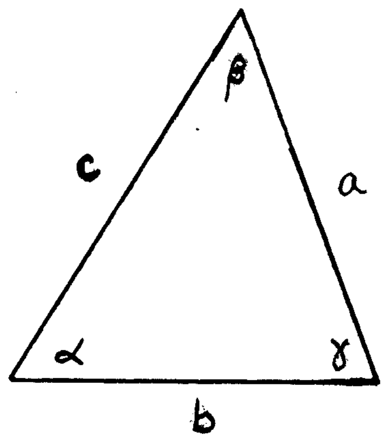
Law of Cosines and Law of Sines
(applicable to any triangle)



Law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$b^2 = c^2 + a^2 - 2ca\cos\beta$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

[Note: for a 90° angle Law of cosines turns into the Pythagorean Theorem since $\cos 90^\circ = 0.000$]

Law of sines:

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Vector Addition using Law of Cosine's and Law of Sines

EXAMPLE

Find the magnitude and direction of the resultant of the two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ shown in Figure 1 if $|\mathbf{F}_1| = 87.2$ pounds and $|\mathbf{F}_2| = 53.8$ pounds.

Solution We complete the parallelogram, as shown in Figure 2. The angle θ is the supplement of 39.7°:

$$\theta = 180° - 39.7° = 140.3°$$

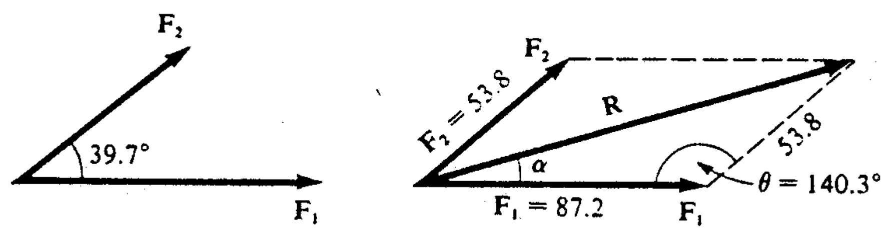


Figure 1 Figure 2

By the law of cosines,

$$\begin{aligned}|\mathbf{R}|^2 &= |\mathbf{F}_1|^2 + |\mathbf{F}_2|^2 - 2|\mathbf{F}_1||\mathbf{F}_2| \cos\theta \\ &= (87.2)^2 + (53.8)^2 - 2(87.2)(53.8)\cos 140.3° \\ &= 17{,}717.34\end{aligned}$$

so $|\mathbf{R}| = 133.1$ pounds. We indicate the direction of $\mathbf{R}$ by finding the angle α by the law of sines.

$$\frac{|\mathbf{F}_2|}{\sin\alpha} = \frac{|\mathbf{R}|}{\sin\theta}$$

$$\sin\alpha = \frac{|\mathbf{F}_2|\sin\theta}{|\mathbf{R}|} = \frac{(53.8)\sin 140.3°}{133.1} = 0.2582$$

Thus, $\alpha = 15.0°$.