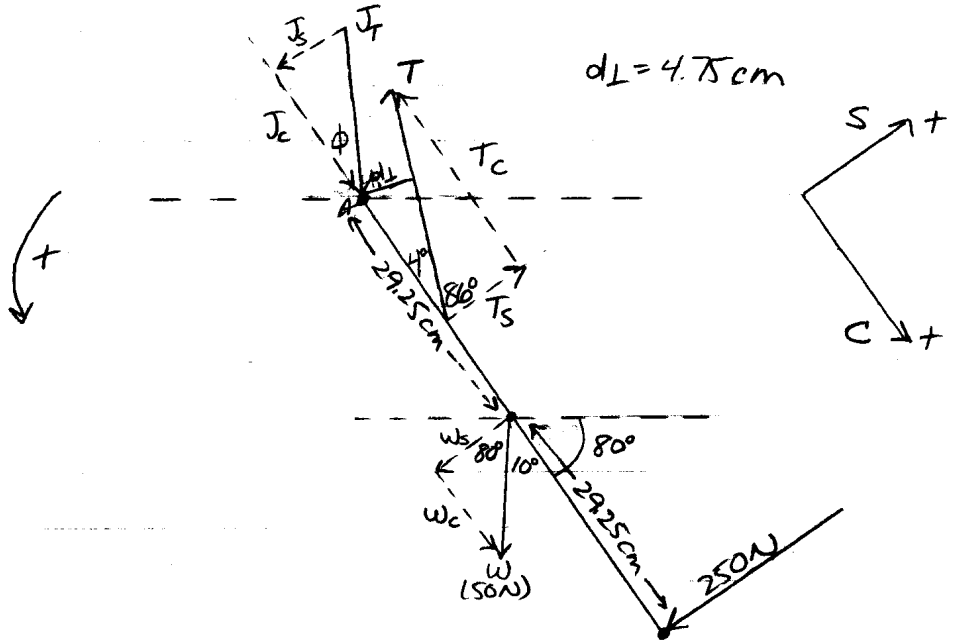
KIN 512 Problem Set #1

a) Draw a complete Free Body Diagram. (FBD)



b) Calculate T (patellar ligament force)

$\circlearrowleft + \sum M_A = 0$

$\Rightarrow \quad T d_L - (W\cos(80°))(29.25\text{cm}) - (250N)(58.5\text{cm}) = 0$

$\Rightarrow \quad T = \dfrac{(50)(\cos(80°))(29.25\text{cm}) + (250N)(58.5\text{cm})}{4.75\text{cm}}$

$\Rightarrow \quad T = 3132.412728\,N \quad \Rightarrow \quad \boxed{T = 3130N}$

c) Calculate compressive and shear components of the joint force J_T.

$+\downarrow \sum F_C = 0$

$\Rightarrow \quad J_C + W_C - T_C = 0$

$\Rightarrow \quad J_C = T\sin 86° - W\sin 80°$

$\Rightarrow \quad J_C = (3132.412728N)(\sin 86) - (50N)(\sin 80)$

$\Rightarrow \quad J_C = 3075.4194N \quad \Rightarrow \quad \boxed{J_C = 3080N}$

$+\uparrow \Sigma F_S = 0$

$-J_S - W_S - 250N + T_S = 0$

$\Rightarrow J_S = T_S - W_S - 250N$

$\Rightarrow J_S = T\cos(86) - W\cos(80) - 250N$

$\Rightarrow J_S = (3132.412728N)\cos(86) - (50N)\cos(80) - 250N$

$\Rightarrow J_S = -40.176342674N \Rightarrow \boxed{J_S = -40.2N}$

d) Calculate the magnitude of J_T and its direction.

$|J_T| = \sqrt{J_S^2 + J_C^2}$

$\Rightarrow |J_T| = \sqrt{(-40.176342674)^2 + (3075.4194)^2}$

$\Rightarrow |J_T| = 3075.6818144N \Rightarrow \boxed{|J_T| = 3080N}$

The direction of J_T is given by ϕ (see FBD)

$\tan\phi = \frac{J_S}{J_C} \Rightarrow \phi = \tan^{-1}\left(\frac{J_S}{J_C}\right)$

$\Rightarrow \phi = \tan^{-1}\left(\frac{-40.176342674}{3075.4194}\right)$

$\Rightarrow \phi = -0.748452043^\circ \Rightarrow \boxed{\phi = -0.75^\circ}$

(ie J_T is at an angle of -0.75° relative to the long axis of the Tibia).

e) Since $J_S = -40.2N$, it means that J_S is directed anteriorly, therefore J_S is sustained by the PCL to counteract net posteriorly directed force. (the PCL provides anteriorly directed force to prevent the Tibia from translating posteriorly).

Note: If you got a negative value for J_S it means that you drew the direction of J_T wrong on your FBD. Rather than showing J_T pointing slightly posteriorly, it actually points slightly anteriorly (as indicated in this solution above). The negative angle ϕ also reflects the same thing. It is OK to keep these negative values; or you could redraw J_T in your FBD and solve for positive values for J_S and ϕ. Either way, the interpretation is the same.

f) Calculate Q (quadriceps muscle force), knowing T and an appropriate T/Q ratio.

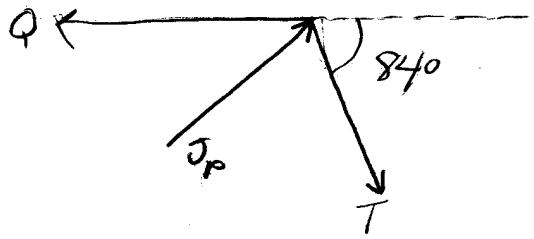
Here we have 80° of knee flexion, so from the literature (from Miller et al (1997) Clinical Biomechanics, 12, 1-7, on Page K22 of the class notes) at 80° of flexion an appropriate T/Q ratio is ~ 0.7 (which is approximately in the mid range of all data shown).

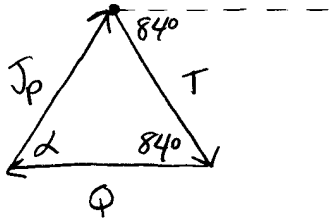
We found $T = 3132.412728\,N$ from part b)

$\therefore \frac{T}{Q} = 0.7 \Rightarrow Q = \frac{T}{0.7} = \frac{3132.412728\,N}{0.7}$

$\Rightarrow Q = 4474.875326\,N \Rightarrow \boxed{Q = 4470\,N}$

g) Complete a Free Body Diagram of the Patella. Compute the Patellofemoral Joint Force (J_P).





Using the law of Cosines to get J_P:

$\Rightarrow J_P^2 = T^2 + Q^2 - 2TQ\cos(84°)$

$\Rightarrow J_P^2 = (3132.412728)^2 + (4474.875326)^2 - 2(T)(Q)\cos(84°)$

$\Rightarrow J_P^2 = 2.690613504 \times 10^7$

$\Rightarrow J_P = 5187.112399\,N \Rightarrow \boxed{J_P = 5190\,N}$

h) Calculate the direction (α) of J_p.

Using the Law of Sines we get:

$$\frac{\sin\alpha}{T} = \frac{\sin 84}{J_p}$$

$$\Rightarrow \sin\alpha = \frac{T \sin 84}{J_p} \Rightarrow \alpha = \sin^{-1}\left(\frac{T \sin 84}{J_p}\right)$$

$$\Rightarrow \alpha = \sin^{-1}\left(\frac{(3132.412728)\sin 84}{(5187.112399)}\right) = \sin^{-1}(0.600575581)$$

$$\Rightarrow \alpha = 36.91113173^\circ \Rightarrow \boxed{\alpha = 36.9^\circ}$$

$\therefore$ J_p is at an angle of 36.9° with respect to Q.