**PART A**

Dimensions in m.

All answers to 3 significant figures.

\[ W = 12 \times 9.8 = 117.6 \text{ N} \]
\[ LLF = 3.5 \times 9.8 = 34.3 \text{ N} \]

\[ \Sigma M_B = 0 = T \times 0.048 - W \times 0.35 - LLF \times 0.15 \]

\[ - T = \frac{117.6 \times 0.35 + 34.3 \times 0.15}{0.048} = 1102.5 \text{ N} = 1100 \text{ N} \]

\[ \Sigma F_x = 0 = T_x + T_{Tx} \]
\[ T_{Tx} = T_x = \frac{1102.5 \cos 30^\circ}{0.048} = 1036.01114 \text{ N} = 1030 \text{ N} \]

\[ \Sigma F_y = 0 = T_y - LLF - W - T_{Ty} \]
\[ T_{Ty} = T_y - LLF - W = 1102.5 \sin 30^\circ - 34.3 - 117.6 \]
\[ = 225 \times 177208 \text{ N} = 225 \text{ N} \]

\[ T = \sqrt{\left(1036.01114 \right)^2 + \left(225 \times 177208 \right)^2} = 1060.19988 \text{ N} = 1060 \text{ N} \]

\[ \Theta = \tan^{-1} \left( \frac{225 \times 177208}{1036.01114} \right) = 12.3625 \text{ N} = 12.3^\circ \]

\( T_{Tx} \) is the compressive component of the joint reaction force.

\( T_{Ty} \) is the shear component of the joint reaction force. This force tends to pull the tibia anterior with respect to the femur and with therefore put tension on the ACL.
PART A (cont)

Using figure 7, Miller et al (1997) and taking an "average" of the different plots, we extrapolated to a knee flexion angle of 0°.

\[ \frac{Q_{TF}}{Q_{TF}} = 1.1 \]

\[ \therefore \frac{T}{Q} = 1.1 \]

\[ \Rightarrow Q = \frac{T}{1.1} = \frac{1102.5}{1.1} = 1002.2727\text{N} \]

\[ J_p = \sqrt{T^2 + Q^2 - 2TQ \cos 30°} \]

\[ = \sqrt{1102.5^2 + (1002.2727)^2 - 2 \times 1102.5 \times 1002.2727 \times \cos 30°} \]

\[ = 553.0909\text{N} \]
\[ \sum F \bigg|_{Axial} = -T_A + \bar{h} F_A + W_A + \dot{T}_A = 0 \]

\[ \dot{T}_A = T_A - \bar{h} F_A - W_A \]

\[ = T \cos 10 - \bar{h} F 5\text{mi} 60 - W 5\text{mi} 60 \]

\[ = 514.5 \times \cos 10 - 34.3 \text{mi} 60 - 117.6 \text{mi} 60 \]

\[ = 375 \times 1343301 \]

\[ \dot{T}_{Axial} = 375 \text{ N} \quad \text{compressive force.} \]
\[ \sum F_{\text{sum}} = 0 = T_s - \bar{L} L F_s - W_s - J_{T_s} \]

\[ \Rightarrow \mathbf{T}_{T_s} = T_s - \bar{L} L F_s - W_s \]

\[ = T_s \sin 10 - \bar{L} L F \cos 60 - W \cos 60 \]

\[ = 5.145 \times 10 - 24.3 \cos 60 - 117.6 \cos 60 \]

\[ = 13.39198741 \]

\[ J_{T_s} = 13.4 \text{ N} \]

The shear applied to the tibia by the femur is in the posterior direction. The tibia applies an anterior force to the femur and will place its ACL slightly in tension. This tension is much less than that applied when the knee is fully extended.

\[ |J_T| = \sqrt{J_{T_a}^2 + J_{T_s}^2} = \sqrt{(375.1393301)^2 + (13.39198741)^2} \]

\[ = 375.3732955 \]

\[ |J_T| = 375 \text{ N} \]

\[ \Theta = \tan^{-1} \frac{13.39198741}{375.1393301} \approx 2.0445406 \]

\[ \Theta = 2.04^\circ \]
For knee flexed 60° the angle between T and Q is 70°.

From Figure 7 Miller et al (1997) a ratio of \(\frac{PTF}{QTF}\) of 0.82 is assumed.

\[\frac{T}{Q} = 0.82\]
\[Q = \frac{T}{0.82} = \frac{514.5}{0.82} = 627.0390244 \approx 627 \text{ N}\]

\[-P = \sqrt{Q^2 + T^2 - 2QT \cos \theta}\]
\[= \sqrt{(627.0390244)^2 + (514.5)^2 - 2 \times 627.0390244 \times 514.5 \cos 70°}\]
\[= 661.4906556 \approx 661 \text{ N}\]
\[ \Sigma M_B = 0 = T_x \cdot 0.042 - 4.5F \cdot 0.15 - W \cdot 0.35 \cos 60 \]

\[ W' = \frac{110.25 \cdot 0.042 - 34.3 \cdot 0.15}{0.35 \cdot 0.5} = 235.2 \text{ N} = 240 \text{ kg} \]

(The moment arm is halved (\( \cos 60 \)) so the weight needs to double to apply the same moment)

Because the same torque is applied to the knee and the patellar tendon force is the same, acting in the same direction, \( \Sigma F_x = 0 \) will be the same and the compressive joint force will be the same

\[ T_{\text{axial}} = 1036.0 \text{ N} \]

\[ = 1026 \text{ N} \]

\[ \Sigma F_y = 0 = T_y - 4.5F - W' - T_{\text{shear}} \]

\[ T_{\text{shear}} = 110.25 \sin 20^\circ - 34.3 - 235.2 \]

\[ = 107.577208 \text{ N} \]

\[ T_{\text{shear}} = 108 \text{ N} \]

This is the force that applies tension to the ACL. The weight attached to the leg is greater but due to the shorter moment arm it is supported by the same patellar tendon force.
This greater weight counteracts the shear force generated by the patella tendon force and the force in the ACL is less than in part A.

\[ T = \sqrt{T_{xc}^2 + T_{Ly}^2} \]

\[ = \sqrt{(1036.01114)^2 + (107.577208)^2} \]

\[ = 1041.581434 \]

\[ T = 1040 \text{ N} \]

\[ \Theta = \tan^{-1}\left(\frac{107.577208}{1036.01114}\right) = 5.98227109 = 5.93^\circ \]

The patella tendon force \( T \) is the same as part A, so the quadriceps muscle force \( Q \) and patellofemoral joint force \( J_p \) will also be the same.

\[ Q = 1000 \text{ N} \]

\[ J_p = 55.3 \text{ N} \]

Loading the knee in this way reduces the tension on the ACL at full extension while still maintaining the same quadriceps muscle force.