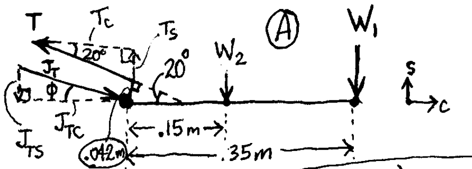


$W_1 = m_1 g = 12\,kg(9.8\,m/s^2) = 117.6\,N$

$W_2 = m_2 g = 3.5\,kg(9.8\,m/s^2) = 34.3\,N$

8.5A problem $|M_1| = 117.6\,N(.35\,m) + 34.3\,N(.15\,m) = \underline{46.305} = \boxed{46.3\,N\cdot m}$ (at full extension)

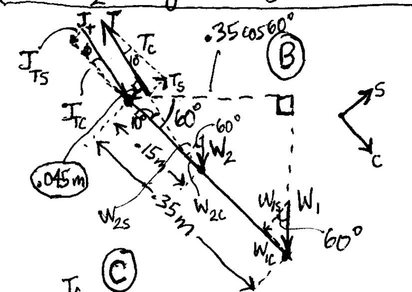
$|M_2| = 117.6\,N(.35\cos 60°) + 34.3\,N(.15\cos 60°) = \underline{23.1525} = \boxed{23.2\,N\cdot m}$ (at 60° flexion)

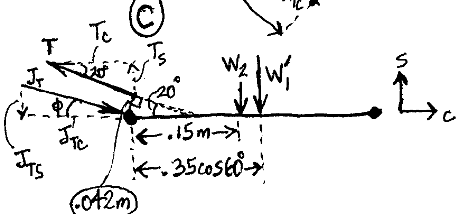
8.5B problem $W_1'(.35\cos 60°) + 34.3\,N(.15) = 46.305\,N\cdot m$

$W_1' = [46.305 - 34.3(.15)]/(.35\cos 60°) = \boxed{235.2\,N}$

$(m_1' = W_1'/g = 235.2/9.8 = 24.0\,kg)$

(so twice the weight at half the distance)





For Figure (A) (full extension, 12 kg mass at .35 m)

$\circlearrowleft \Sigma M_K = 0 \Rightarrow T(.042) - M_1 = 0 \Rightarrow T = M_1/.042 = 46.305/.042$

$T = \underline{1102.5\,N} = \boxed{1100\,N}$

$\xrightarrow{+} \Sigma F_C = 0 \Rightarrow J_{TC} - T_C = 0 \Rightarrow J_{TC} = T_C = (1102.5)\cos 20° = \underline{1036.011114\,N}$

$J_{TC} = \boxed{1040\,N}$

$\overset{+}{\uparrow}\ \Sigma F_S = 0 \Rightarrow -J_{TS} + T_S - W_1 - W_2 = 0 \Rightarrow J_{TS} = T_S - W_1 - W_2$

$J_{TS} = (1102.5)\sin 20° - 117.6 - 34.3 = \underline{225.177208} = \boxed{225\,N}$

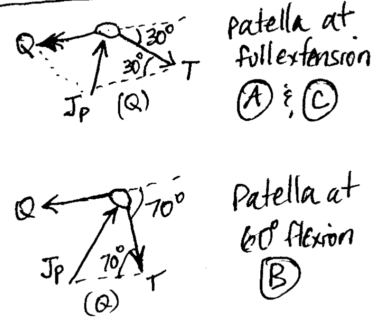
Since J_{TS} points posteriorly: $\boxed{\text{ACL sustains}}$ *goes here

use $T/Q = 1.1$ at full extension (Miller et al, 1997; Eijden et al., 1986)

$Q = T/1.1 = 1102.5/1.1 = \underline{1002.272727\,N}$

$J_P = \sqrt{T^2 + Q^2 - 2TQ\cos 30°} = 553.2909599 = \boxed{553\,N}$

$$* \left\{ \begin{array}{l} J_T = \sqrt{J_{TS}^2 + J_{TC}^2} = 1060.199889 = \boxed{1060\,N} \\ \phi = \tan^{-1}(J_{TS}/J_{TC}) = 12.26252409° = \boxed{12.3°} \end{array} \right\}$$



For Figure (B) (60° flexion, 12 kg mass at .35cos60° m)

$\circlearrowleft \Sigma M_K = 0 \Rightarrow T(.045) - M_2 = 0 \Rightarrow T = M_2/.045 = 23.1525/.045 = 514.5\,N = \boxed{515\,N}$

$\overset{+}{\searrow} \Sigma F_C = 0 \Rightarrow J_{TC} - T_C + W_{2C} + W_{1C} = 0 \Rightarrow J_{TC} = T_C - W_{2C} - W_{1C} = 514.5\cos 10° - (34.3)\sin 60° - (117.6)\sin 60° = \underline{375.1343301\,N} = \boxed{375\,N}$

$\overset{+}{\nearrow} \Sigma F_S = 0 \Rightarrow -J_{TS} + T_S - W_{2S} - W_{1S} = 0 \Rightarrow J_{TS} = T_S - W_{2S} - W_{1S} = 514.5\sin 10° - 34.3\cos 60° - 117.6\cos 60°$

$J_{TS} = \underline{13.39198741\,N} = \boxed{13.4\,N}$ $\boxed{\text{since } J_{TS} \text{ points posteriorly ACL sustains}}$

$J_T = \sqrt{J_{TS}^2 + J_{TC}^2} = 375.3732955 = \boxed{375\,N}$ $\quad \phi = \tan^{-1}(J_{TS}/J_{TC}) = 2.044544011 = \boxed{2.04°}$

use $T/Q = 0.8$ at 60° flexion (Miller et al., 1997) $\Rightarrow Q = T/0.8 = 514.5/0.8 = \underline{643.125\,N}$

$J_P = \sqrt{T^2 + Q^2 - 2TQ\cos 70°} = 672.2941481 = \boxed{672\,N}$

For Figure (C) (full extension, 24 kg mass at .35cos60° m) $\circlearrowleft \Sigma M_K = 0 \Rightarrow T(.042) - M_1 = 0 \Rightarrow$ same as (A) $\boxed{T = 1100\,N}$

$\overset{+}{\uparrow} \Sigma F_S = 0 \Rightarrow -J_{TS} + T_S - W_2 - W_1' = 0 \Rightarrow J_{TS} = T_S - W_2 - W_1' \Rightarrow \boxed{108\,N}$

$\xrightarrow{+} \Sigma F_C = 0 \Rightarrow J_{TC} - T_C = 0 \Rightarrow J_{TC} = T_C = \boxed{1040\,N}$ same as (A)

like (A): $\boxed{\text{ACL}} \leftarrow J_{TS} = (1102.5)\sin 20° - 34.3 - 235.2 = \underline{107.577208}$

$J_T = \sqrt{J_{TS}^2 + J_{TC}^2} = 1041.581434 = \boxed{1040\,N}$, $\phi = \tan^{-1}(J_{TS}/J_{TC}) = 5.928227109 = \boxed{5.93°}$, J_P = same as (A) = $\boxed{553\,N}$

The moral of the story is that Figure C represents a solution that is easier on the ACL than Figure A. The quads are exerting the same forces in each case; the patellofemoral joint force is the same in each case; yet the shear component of the tibiofemoral force is less than half as large in Figure C as in Figure A (108 N vs. 225 N). This results from lifting a heavier weight at a reduced moment arm. The heavier weight counteracts more of the shear component of the patellar tendon force and hence reduces the force in the ACL. Cool, huh? A logical extension of this problem is to see what happens when this new weight is lowered to 60° of flexion.