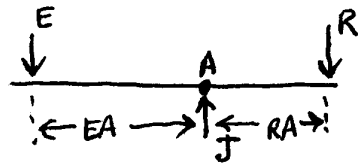
KIN 412/512 EXAMPLE LEVER PROBLEM SOLUTIONS

1a.

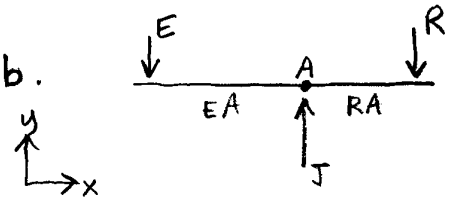


$R = 100\,N$ $RA = 60\,cm$ $EA = 85\,cm$ Find E & J

$\circlearrowleft \Sigma M_A = 0 \Rightarrow E(EA) - R(RA) = 0 \Rightarrow E = \frac{R(RA)}{EA} = \frac{100(60)}{85} = \boxed{70.6\,N}$

$\Sigma F = 0 \Rightarrow -E + J - R = 0 \Rightarrow J = E + R = 70.6 + 100 = \boxed{170.6\,N}$

1.b.



$E = 120\,N$ $EA = 1.1\,m$ $RA = 0.8\,m$ Find R & J

$\circlearrowleft \Sigma M_A = 0 \Rightarrow E(EA) - R(RA) = 0$

$R = \frac{E(EA)}{RA} = \frac{120(1.1)}{0.8} = \boxed{165\,N}$

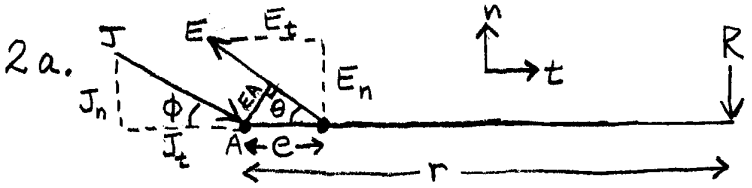
$\Sigma F_y = 0 \Rightarrow J - E - R = 0 \Rightarrow J = E + R = 120 + 165 = \boxed{285\,N}$

1.c. same diagram as above $E = 250\,lb$ $R = 400\,lb$ $EA = 10''$ Find RA & J

$\Sigma M_A = 0 \Rightarrow E(EA) - R(RA) = 0 \Rightarrow RA = \frac{E(EA)}{R} = \frac{250(10)}{400} = \boxed{6.25''}$

$\Sigma F_y = 0 \Rightarrow J - E - R = 0 \Rightarrow J = E + R = 250 + 400 = \boxed{650\,lb}$

2a.



$R = 50\,lb$ $r = 20''$

$e = 2''$ $\theta = 20^\circ$

Find E, J, ϕ

$\circlearrowleft \Sigma M_A = 0 \Rightarrow E_n(e) - R(r) = 0 \Rightarrow E_n = \frac{R \cdot r}{e} = \frac{50(20)}{2} = \underline{500\,lb}$

Also: $E_n = E\sin\theta \Rightarrow E = \frac{E_n}{\sin\theta} = \frac{500}{\sin 20^\circ} = 1461.9022 = \boxed{1462\,lb}$

$\Sigma F_t = 0 \Rightarrow J_t - E_t = 0 \Rightarrow J_t = E_t = E\cos\theta = 1461.9022\cos 20^\circ =$

$= \underline{1373.73871\,lb}$

$\Sigma F_n = 0 \Rightarrow -J_n + E_n - R = 0 \Rightarrow J_n = E_n - R = 500 - 50 = \underline{450\,lb}$

$J = \sqrt{J_t^2 + J_n^2} = \sqrt{(1373.7\ldots)^2 + (450)^2} = 1445.564956 = \boxed{1446\,N}$

$\phi = \tan^{-1}\left(\frac{J_n}{J_t}\right) = \tan^{-1}\left(\frac{450}{1373.7\ldots}\right) = 18.1374097 = \boxed{18.1^\circ}$

2b. (same diagram as 2a.) Find R, J, ϕ

$E = 2500\ N \quad e = 10\ cm \quad \theta = 15^\circ \quad r = 19\ cm$

$\circlearrowleft \Sigma M_A = 0 \Rightarrow E_n(e) - R(r) = 0 \Rightarrow R = \frac{E_n \cdot e}{r} = \frac{E\sin\theta \cdot e}{r}$

$R = \frac{2500 \cdot \sin 15^\circ (10)}{19} = 340.5513751 = \boxed{341\ N}$

$\Sigma F_t = 0 \Rightarrow J_t - E_t = 0 \Rightarrow J_t = E_t = E\cos\theta = 2500\cos 15^\circ = \underline{2414.814566\ N}$

$\Sigma F_n = 0 \Rightarrow -J_n + E_n - R = 0 \Rightarrow J_n = E_n - R = E\sin\theta - R = 2500\sin 15^\circ - 340.55\ldots$

$= 640.0476128 - 340.5513751 = \underline{306.4962376\ N}$

$J = \sqrt{J_t^2 + J_n^2} = \sqrt{(2414.8\ldots)^2 + (306.49\ldots)^2} = 2434.187612 = \boxed{2434\ N}$

$\phi = \tan^{-1}\left(\frac{J_n}{J_t}\right) = \tan^{-1}\left(\frac{306.49\ldots}{2414.8\ldots}\right) = 7.233492271 = \boxed{7.2^\circ}$

2c. (same diagram as 2a.)

$R = 150\ N,\ EA = e\sin\theta = 4.5\ cm,\ \theta = 25^\circ,\ r = 50\ cm,$ Find E, J, ϕ

$\circlearrowleft \Sigma M_A = 0 \Rightarrow E(EA) - R(r) = 0 \Rightarrow E = \frac{R \cdot r}{EA} = \frac{150 \cdot 50}{4.5} = 1666.\overline{6}\ldots = \boxed{1667\ N}$

$\Sigma F_t = 0 \Rightarrow J_t - E_t = 0 \Rightarrow J_t = E_t = E\cos\theta = (1666.6\ldots)\cos 25^\circ$

$= \underline{1510.512978\ N}$

$\Sigma F_n = 0 \Rightarrow -J_n + E_n - R = 0 \Rightarrow J_n = E_n - R = E\sin\theta - R = (1666.6\ldots)\sin 25^\circ - 150$

$= 704.3637696 - 150 = \underline{554.3637696\ N}$

$J = \sqrt{J_t^2 + J_n^2} = \sqrt{(1510.5\ldots)^2 + (554.3\ldots)^2} = 1609.027236 = \boxed{1609\ N}$

$\phi = \tan^{-1}\left(\frac{J_n}{J_t}\right) = \tan^{-1}\left(\frac{554.3\ldots}{1510.5\ldots}\right) = 20.15332088 = \boxed{20.2^\circ}$