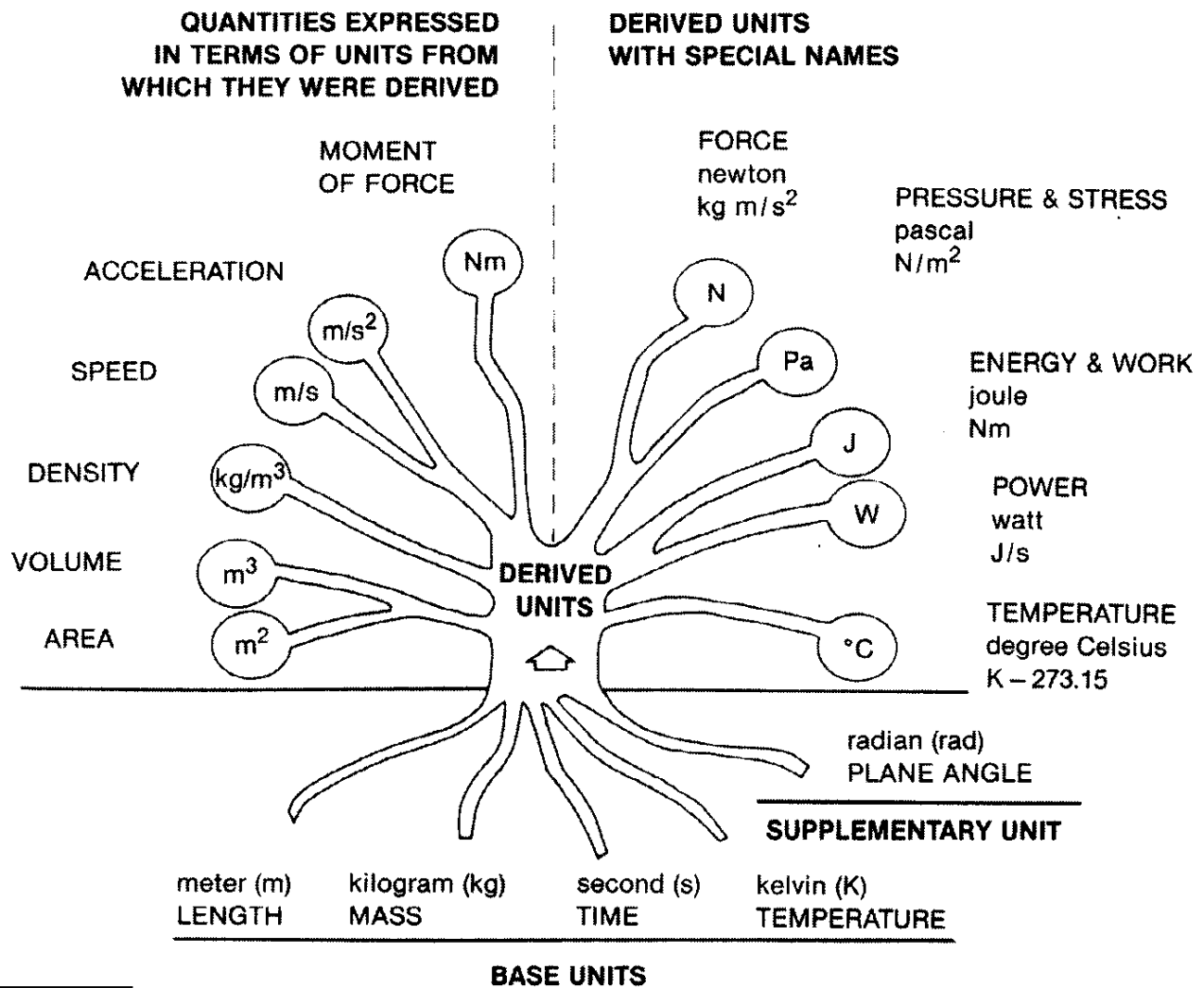


# KIN 412/512 Biomechanics of the Skeletal System

## Introduction



**FIG. 1-1**

The International System of Units.

from Nordin & Frankel (2001).

**Length**

$$1 \text{ centimeter (cm)} = 0.01 \text{ meter (m)}$$

$$1 \text{ inch (in)} = 0.0254 \text{ m}$$

$$1 \text{ foot (ft)} = 0.3048 \text{ m}$$

$$1 \text{ yard (yd)} = 0.9144 \text{ m}$$

$$1 \text{ mile} = 1609 \text{ m}$$

$$1 \text{ angstrom (\AA)} = 10^{-10} \text{ m}$$

**Time**

$$1 \text{ minute (min)} = 60 \text{ second (s)}$$

$$1 \text{ hour (h)} = 3600 \text{ s}$$

$$1 \text{ day (d)} = 86400 \text{ s}$$

**Mass**

$$1 \text{ pound mass (lbm)} = 0.4536 \text{ kilogram (kg)}$$

$$1 \text{ slug} = 14.59 \text{ kg}$$

**Force**

$$1 \text{ kilogram force (kgf)} = 9.807 \text{ newton (N)}$$

$$1 \text{ pound force (lbf)} = 4.448 \text{ N}$$

$$1 \text{ dyne (dyn)} = 10^{-5} \text{ N}$$

**Pressure and Stress**

$$1 \text{ kg/ m-s}^2 = 1 \text{ N/m}^2 = 1 \text{ Pascal (Pa)}$$

$$1 \text{ lbf / in}^2 \text{ (psi)} = 6896 \text{ Pa}$$

$$1 \text{ lbf / ft}^2 \text{ (psf)} = 92966 \text{ Pa}$$

$$1 \text{ dyn / cm}^2 = 0.1 \text{ Pa}$$

**Moment (Torque)**

$$1 \text{ dyn-cm} = 10^{-7} \text{ N-m}$$

$$1 \text{ lbf-ft} = 1.356 \text{ N-m}$$

**Work and Energy**

$$1 \text{ kg-m}^2 / \text{s}^2 = 1 \text{ N-m} = 1 \text{ Joule (J)}$$

$$1 \text{ dyn-cm} = 1 \text{ erg} = 10^{-7} \text{ J}$$

$$1 \text{ lbf-ft} = 1.356 \text{ J}$$

**Power**

$$1 \text{ kg- m}^2 / \text{s}^3 = 1 \text{ J/s} = 1 \text{ Watt (W)}$$

$$1 \text{ horsepower (hp)} = 550 \text{ lbf-ft/s} = 746 \text{ W}$$

**Plane Angle**

$$1 \text{ degree (}^\circ\text{)} = \pi / 180 \text{ radian (rad)}$$

$$1 \text{ revolution (rev)} = 360^\circ$$

$$1 \text{ rev} = 2\pi \text{ rad} = 6.283 \text{ rad}$$

**Temperature**

$$^\circ\text{C} = ^\circ\text{K} - 273.2$$

$$^\circ\text{C} = 5 (^\circ\text{F} - 32)/9$$

Reprinted with permission from Özkaya, N., & Nordin, M. (1999). *Fundamentals of Biomechanics: Equilibrium, Motion, and Deformation* (2nd ed.) New York: Springer-Verlag, p. 11.

# Metric mix-up doomed Mars Climate Orbiter

## Failure to convert led to navigation error

BY MATTHEW FORDAHL  
THE ASSOCIATED PRESS

LOS ANGELES — The \$125 million spacecraft that was destroyed on a mission to Mars was probably doomed by the embarrassing failure to convert English units of measurement to metric ones, NASA said Thursday.

## MIX-UP: Orbiter doomed by mistake

From Page A1

just as it was about to go into orbit around the Red Planet.

Agency officials said the mistake somehow escaped what is supposed to be a rigorous error-checking process. A report is expected in mid-November, which would be soon enough to fix any possible similar problems with a spacecraft already en route to Mars.

"It does not make us feel good that this happened," said Tom Gavin of NASA's Jet Propulsion Laboratory. "This mixup has caused us to look at our entire end-to-end process. We will get to the bottom of this."

The Mars Climate Orbiter flew too close to Mars and is believed to have broken apart or burned up in the atmosphere.

NASA said the English-vs.-metric mix-up apparently caused the navigation error. The company that built the spacecraft acknowledged it made the error.

The mistake was particularly embarrassing because the spacecraft had successfully flown 416 million miles over 9½ months before its disappearance Sept. 23

Please see **MIX-UP**, Page A4

"This is going to be the cautionary tale that is going to be embedded into introductions to the metric system in elementary school and high school and college physics till the end of time," said John Pike, director of space policy at the Federation of American Scientists in Washington.

Lockheed's reaction was equally blunt.

"The reaction is disbelief," said Noel Hinners, vice president for flight systems at Lockheed Martin Astronautics in Denver. "It can't be something that simple that could cause this to happen."

The laboratory said that its preliminary findings showed that Lockheed Martin Astronautics in Colorado submitted acceleration data in English units of pounds of force instead of the metric unit called newtons. At the laboratory, the numbers were entered into a computer that assumed metric measurements.

"In our previous Mars missions, we have always used metric," Gavin said.

The numbers were used in figuring the force of thruster firings used by the spacecraft to adjust its position.

The bad numbers had been used ever since the spacecraft's launch last December, but the effect was so small that it went unnoticed. The difference added up over the months as the spacecraft journeyed toward Mars.

Gavin said he does not expect the error to affect NASA's relationship with Lockheed Martin, which has built several probes for the space agency.

Lockheed Martin acknowledged the mistake.

"We should have had them in metric units," Hinners said.

The Mars Climate Orbiter was on a mission to study the Red Planet's weather and look for signs of water — information key to understanding whether life ever existed or can exist there. It carried cameras along with equipment for measuring temperature, dust, water vapor and clouds.

The orbiter's sibling spacecraft, Mars Polar Lander, is set to arrive Dec. 3. Gavin said investigators are trying to determine whether NASA made the same mistake with that spacecraft. The problem could be fixed if it did occur.

The Mars Polar Lander will study Mars' climate history and weather with the goal of finding what happened to water on the planet. It is equipped with a robotic arm that will collect samples for testing inside the spacecraft.

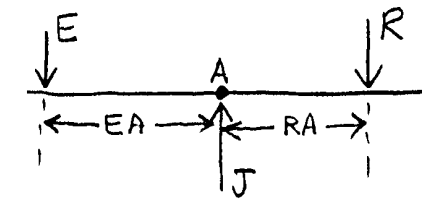
# Static equilibrium problems (examples)

1. See-saw :

$$\Sigma M_A = 0 \quad (\oplus)$$

$$E(EA) - R(RA) = 0$$

$$E(EA) = R(RA)$$



$$\Sigma F_x = 0$$

(no x forces here)

$$\Sigma F_y = 0 \Rightarrow J - E - R = 0$$

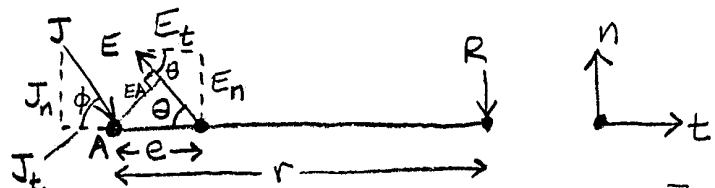
$$J = E + R$$

a. Given  $R = 100 \text{ N}$   $RA = 60 \text{ cm}$   $EA = 85 \text{ cm}$   
Find  $E$  and  $J$

b. Given  $E = 120 \text{ N}$   $EA = 1.1 \text{ m}$   $RA = 0.8 \text{ m}$   
Find  $R$  and  $J$

c. Given  $E = 250 \text{ lb}$   $R = 400 \text{ lb}$   $EA = 10 \text{ in.}$   
Find  $RA$  and  $J$

2. Internal body lever :



$$\Sigma M_A = 0 \quad (\oplus)$$

$$E_n \cdot e - R \cdot r = 0$$

$$\left[ \begin{array}{ll} E_n = E \sin \theta & E_t = E \cos \theta \\ J_n = J \sin \phi & J_t = J \cos \phi \end{array} \right]$$

$$\Sigma F_t = 0 \Rightarrow J_t - E_t = 0 \Rightarrow J_t = E_t$$

$$\Sigma F_n = 0 \Rightarrow -J_n + E_n - R = 0 \Rightarrow J_n = E_n - R$$

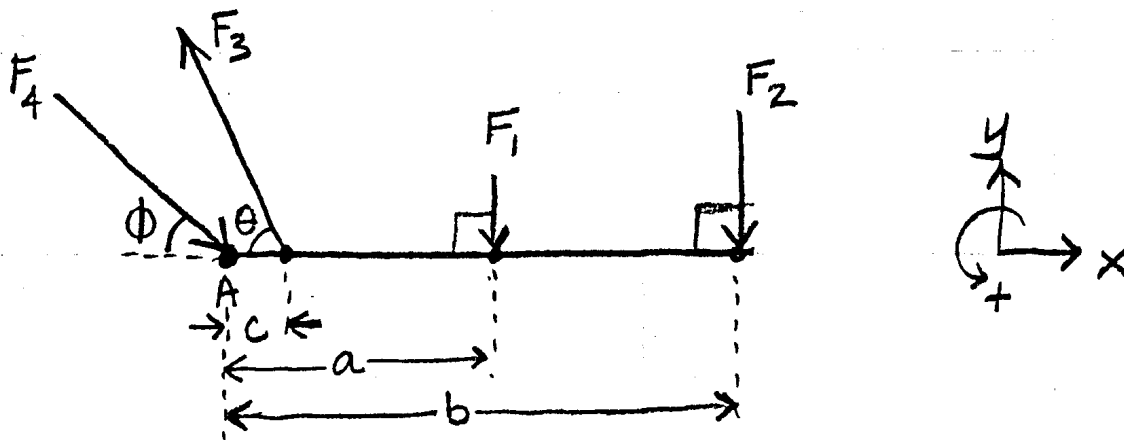
$$J = \sqrt{J_n^2 + J_t^2} \quad \phi = \tan^{-1} \left( \frac{J_n}{J_t} \right)$$

a. Given  $R = 50 \text{ lb}$ ,  $r = 20 \text{ in.}$ ,  $e = 2 \text{ in.}$ ,  $\theta = 20^\circ$   
Find  $E$ ,  $J$ ,  $\phi$  (Note: leave units as lb and in)

b. Given  $E = 2500 \text{ N}$ ,  $e = 10 \text{ cm}$ ,  $\theta = 15^\circ$ ,  $r = 19 \text{ cm}$   
Find  $R$ ,  $J$ ,  $\phi$

c. Given  $R = 150 \text{ N}$ ,  $EA = e \sin \theta = 4.5 \text{ cm}$ ,  $\theta = 25^\circ$ ,  $r = 50 \text{ cm}$   
Find  $E$ ,  $J$ ,  $\phi$

# Solution to example muscle & joint force problem.



Given  $F_1 = 3 \text{ lb}$ ,  $F_2 = 25 \text{ lb}$ ,  $a = 8''$ ,  
 $b = 16''$ ,  $c = 2''$ ,  $\theta = 75^\circ$

Find  $F_3$ ,  $F_4$ ,  $\phi$

Step 1: sum the moments about axis A and set them equal to zero (equilibrium) This will solve for muscle force  $F_3$ .

$$\textcircled{+} \Sigma M_A = 0 \Rightarrow F_3 c \sin \theta - F_1 a - F_2 b = 0$$

$$F_3 = (F_1 a + F_2 b) / (c \sin \theta)$$

$$= (3 \cdot 8 + 25 \cdot 16) / (2 \sin 75^\circ) =$$

$$= 424 / 1.9318517 = \boxed{219.5} \text{ lb}$$

Step 2: sum the forces and set them equal to zero (equilibrium) for  $x$  and  $y$  components separately. This will solve for the joint contact force components  $F_4 \cos \phi$  and  $F_4 \sin \phi$ .

$$\rightarrow + \quad \Sigma F_x = 0 \Rightarrow F_4 \cos \phi - F_3 \cos \theta = 0$$

$$F_4 \cos \phi = F_3 \cos \theta = 219.47855 \cos 75^\circ = \underline{\hspace{2cm}} \text{ lb}$$

(Note: this is the x component of  $F_4$ )

$$\begin{matrix} + \\ \uparrow \end{matrix} \quad \Sigma F_y = 0 \Rightarrow -F_4 \sin \phi + F_3 \sin \theta - F_1 - F_2 = 0$$

$$F_4 \sin \phi = F_3 \sin \theta - F_1 - F_2 \\ = 219.47855 \sin 75^\circ - 3 - 25 = \underline{\hspace{2cm}} \text{ lb}$$

(Note: this is the y component of  $F_4$ .)

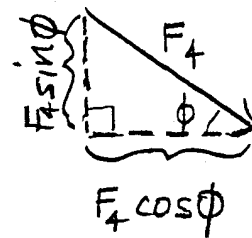
Step 3: Use the pythagorean theorem and inverse tangent to solve for joint contact force magnitude ( $F_4$ ) and direction ( $\phi$ ).

$$F_4^2 = (F_4 \cos \phi)^2 + (F_4 \sin \phi)^2$$

$$F_4 = \sqrt{(F_4 \cos \phi)^2 + (F_4 \sin \phi)^2}$$

$$= \sqrt{56.805229^2 + 184^2}$$

$$= \underline{\hspace{2cm}} \text{ lb}$$



$$\tan \phi = \frac{F_4 \sin \phi}{F_4 \cos \phi} \Rightarrow \phi = \tan^{-1} \left( \frac{F_4 \sin \phi}{F_4 \cos \phi} \right)$$

$$\phi = \tan^{-1} \left( \frac{184}{56.805229} \right) = \underline{\hspace{2cm}}^\circ$$

Concept: Because the muscle has a much smaller moment arm ( $c \sin \theta$ ) than that of the external weight being lifted ( $b$ ), the muscle force has to be much larger than the weight ( $F_3 \approx 9F_2$  here). The large muscle force also tends to compress the bones of the joint together, creating a large joint contact force as well ( $F_4 \approx 8F_2$  here).