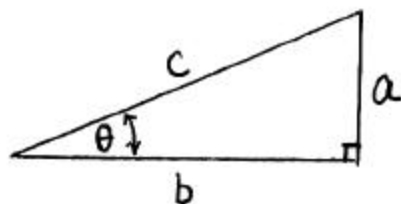


Elementary Trigonometry Reference SheetRight Triangle

a = "opposite side"
(side opposite the angle θ)

b = "adjacent side"
(side adjacent to the angle θ)

c = "hypotenuse" (longest side,
side opposite the 90° angle)

Equations to memorize:

$$1. \sin \theta = \frac{\text{opp.}}{\text{hyp.}} = \frac{a}{c} \quad (\text{sine})$$

$$2. \cos \theta = \frac{\text{adj.}}{\text{hyp.}} = \frac{b}{c} \quad (\text{cosine})$$

$$3. \tan \theta = \frac{\text{opp.}}{\text{adj.}} = \frac{a}{b} \quad (\text{tangent})$$

4. Pythagorean Theorem:

$$c^2 = a^2 + b^2$$

Note: in vector problems, c is known as the "resultant" vector, b is the "horizontal component" and a is the "vertical component" of the vector.

Common Applications

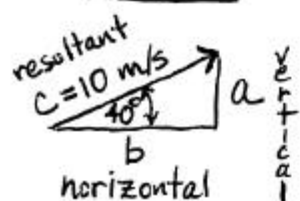
Type A problem: Given a triangle with c and θ known, find a and b . (E.g. given a vector with magnitude, c , and direction θ known, find the horizontal and vertical components of the vector.)

$$\text{Equation 1: } \sin \theta = \frac{a}{c} \Rightarrow a = c \sin \theta$$

$$\text{Equation 2: } \cos \theta = \frac{b}{c} \Rightarrow b = c \cos \theta$$

→ Example: Say a shotputter releases the shot so that it is moving at a speed of 10 m/s (meters/second) at an angle of 40° to the horizontal. Find the horizontal and vertical components of the velocity of release of the shot.

→ Solution:



Note that $\sin 40^\circ = .6428$, $\cos 40^\circ = .7660$

$$a = c \sin 40^\circ = \underline{\hspace{2cm}} = \boxed{\hspace{1cm} \text{m/s} \hspace{1cm}}$$

$$b = c \cos 40^\circ = \underline{\hspace{2cm}} = \boxed{\hspace{1cm} \text{m/s} \hspace{1cm}}$$

Type B problem: Given a triangle with a and b known, find c and θ . (E.g. given a vector having horizontal and vertical components known, find its magnitude, c , and its direction θ .)

Equation 4: $c^2 = a^2 + b^2 \Rightarrow c = \underline{\hspace{2cm}}$

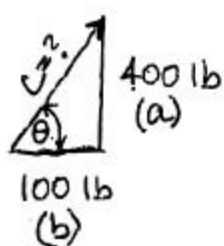
Equation 3: $\tan \theta = \frac{a}{b} \Rightarrow \theta = \tan^{-1}(\tan \theta) = \tan^{-1}\left(\frac{a}{b}\right)$

Note: $\tan^{-1}$ means "inverse tangent" or "the angle whose tangent is". It is commonly referred to as "arctan".

For example: $\tan 45^\circ = 1.0$ (the tangent of 45° is 1.0), therefore $\tan^{-1}(1.0) = 45^\circ$ (the inverse tangent of 1.0 is 45° , or "the angle which has a tangent of 1.0 is 45° "). $\tan$ and $\tan^{-1}$ are related in a similar way as square and square root.

→ Example: During a running stride, the ground exerts an upward and forward force on the runner. If the horizontal force is 100 lb and the vertical force is 400 lb, what is the magnitude of the resultant force vector acting on the runner (from the ground) and in which direction does it act?

→ Solution:



$$c^2 = a^2 + b^2 \Rightarrow c = \sqrt{a^2 + b^2} = \underline{\hspace{2cm}} = \boxed{\hspace{1cm} \text{lb} \hspace{1cm}}$$

$$\theta = \tan^{-1} \frac{a}{b} = \tan^{-1}\left(\frac{400}{100}\right) = \underline{\hspace{2cm}} = \boxed{\hspace{1cm} ^\circ \hspace{1cm}}$$

Practice Algebra Problems

1. $3x = 17$

2. $\frac{x}{5} = 13$

3. $\frac{x}{3} + 21 = 14$

4. $2(5-x) = 10(20x-7)$

5. $(\frac{15}{x} + 3)7 - 9x = 0$

6. $5x^2 = 17$

7. $\frac{1}{4}x^2 - \frac{3}{5} = \frac{5}{7}$

8. $\frac{3x}{8} - \frac{1}{3} = 15$

9. $d = \frac{1}{2}at^2$ solve for t in terms of a and d .

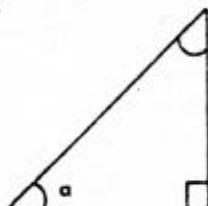
10. $v_f^2 = 2ad$ solve for d in terms of v_f and a .

Practice Trig. Problems

APPENDIXES 413

PROBLEMS*

1. In the following triangle, label the sides (opposite, hypotenuse, and adjacent) with respect to angle a .

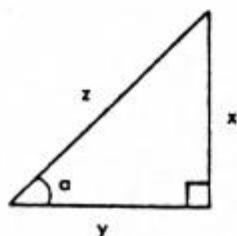


2. In the following triangle, complete the appropriate ratios.

$$\sin a = \frac{\quad}{\quad}$$

$$\cos a = \frac{\quad}{\quad}$$

$$\tan a = \frac{\quad}{\quad}$$



3. Using Table B-1, find the following.

$$\sin 30^\circ$$

$$\cos 30^\circ$$

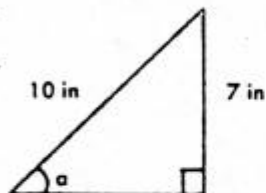
$$\tan 40^\circ$$

$$\sin 60^\circ$$

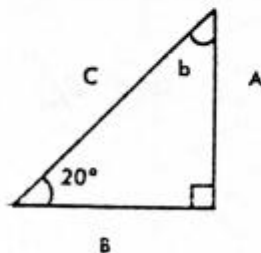
$$\cos 60^\circ$$

$$\tan 60^\circ$$

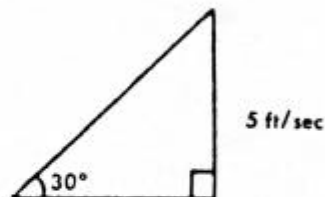
4. In the following triangle, find the value of angle a .



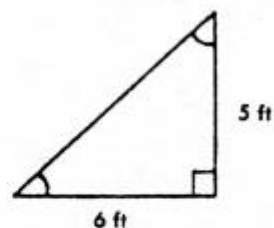
5. If angle a is 20 degrees and side A is 10 inches long, find sides B , and C and angle b .



6. Find the two unknown sides and one unknown angle in the following triangle.



7. A woman walks 8 miles east, turns left, and walks 16 miles north. What is her resultant displacement?
8. The side of a mountain makes a 20-degree angle with the horizontal. If a man walks 1 mile up the mountain, how much has he increased his elevation?
9. A football is kicked with a resultant velocity of 12 m/sec at a 20-degree angle to the ground. What are the horizontal and vertical components of the velocity?
10. Without using trigonometric functions, determine the length of the hypotenuse (also called the resultant side) in the following triangle.



11. A man walks 36 km east and then 10.4 km north. What is his resultant displacement?
12. An automobile travels on a road 50 degrees north of east. If the car goes 17 miles, what are the north and east components of the displacement?
13. The side of a mountain makes a 30-degree angle with the horizontal surface of the earth. If a man walks 2 miles up the mountain side, what is his vertical elevation above the horizontal surface?

TABLE V.1
TRIGONOMETRIC FUNCTIONS

Radian	Degree	Sine	Cosine	Tangent	Radian	Degree	Sine	Cosine	Tangent
.000	0	0.0000	1.0000	0.0000	.803	46	0.7193	0.6947	1.0355
.017	1	0.0175	0.9998	0.0175	.820	47	0.7314	0.6820	1.0724
.035	2	0.0349	0.9994	0.0349	.838	48	0.7431	0.6691	1.1106
.052	3	0.0523	0.9986	0.0524	.855	49	0.7547	0.6561	1.1504
.070	4	0.0698	0.9976	0.0699	.873	50	0.7660	0.6428	1.1918
.087	5	0.0872	0.9962	0.0875	.890	51	0.7771	0.6293	1.2349
.105	6	0.1045	0.9945	0.1051	.908	52	0.7880	0.6157	1.2799
.122	7	0.1219	0.9925	0.1228	.925	53	0.7986	0.6018	1.3270
.140	8	0.1392	0.9903	0.1405	.942	54	0.8090	0.5878	1.3764
.157	9	0.1564	0.9877	0.1584	.960	55	0.8192	0.5736	1.4281
.175	10	0.1736	0.9848	0.1763	.977	56	0.8290	0.5592	1.4826
.192	11	0.1908	0.9816	0.1944	.995	57	0.8387	0.5446	1.5399
.209	12	0.2079	0.9781	0.2126	1.012	58	0.8480	0.5299	1.6003
.227	13	0.2250	0.9744	0.2309	1.030	59	0.8572	0.5150	1.6643
.244	14	0.2419	0.9703	0.2493	1.047	60	0.8660	0.5000	1.7321
.262	15	0.2588	0.9659	0.2679	1.065	61	0.8746	0.4848	1.8040
.279	16	0.2756	0.9613	0.2867	1.082	62	0.8829	0.4695	1.8807
.297	17	0.2924	0.9563	0.3057	1.100	63	0.8910	0.4540	1.9626
.314	18	0.3090	0.9511	0.3249	1.117	64	0.8988	0.4384	2.0503
.332	19	0.3256	0.9455	0.3443	1.134	65	0.9063	0.4226	2.1445
.349	20	0.3420	0.9397	0.3640	1.152	66	0.9135	0.4067	2.2460
.367	21	0.3584	0.9336	0.3839	1.169	67	0.9205	0.3907	2.3559
.384	22	0.3746	0.9272	0.4040	1.187	68	0.9272	0.3746	2.4751
.401	23	0.3907	0.9205	0.4245	1.204	69	0.9336	0.3584	2.6051
.419	24	0.4067	0.9135	0.4452	1.222	70	0.9397	0.3420	2.7475
.436	25	0.4226	0.9063	0.4663	1.239	71	0.9455	0.3256	2.9042
.454	26	0.4384	0.8988	0.4877	1.257	72	0.9511	0.3090	3.0777
.471	27	0.4540	0.8910	0.5095	1.274	73	0.9563	0.2924	3.2709
.489	28	0.4695	0.8829	0.5317	1.292	74	0.9613	0.2756	3.4874
.506	29	0.4848	0.8746	0.5543	1.309	75	0.9659	0.2588	3.7321
.524	30	0.5000	0.8660	0.5774	1.326	76	0.9703	0.2419	4.0108
.541	31	0.5150	0.8572	0.6009	1.344	77	0.9744	0.2250	4.3315
.559	32	0.5299	0.8480	0.6249	1.361	78	0.9781	0.2079	4.7046
.576	33	0.5446	0.8387	0.6494	1.379	79	0.9816	0.1908	5.1446
.593	34	0.5592	0.8290	0.6745	1.396	80	0.9848	0.1736	5.6713
.611	35	0.5736	0.8192	0.7002	1.414	81	0.9877	0.1564	6.3138
.628	36	0.5878	0.8090	0.7265	1.431	82	0.9903	0.1392	7.1154
.646	37	0.6018	0.7986	0.7536	1.449	83	0.9925	0.1219	8.1443
.663	38	0.6157	0.7880	0.7813	1.466	84	0.9945	0.1045	9.5144
.681	39	0.6293	0.7771	0.8098	1.484	85	0.9962	0.0872	11.43
.698	40	0.6428	0.7660	0.8391	1.501	86	0.9976	0.0698	14.30
.716	41	0.6561	0.7547	0.8693	1.518	87	0.9986	0.0523	19.08
.733	42	0.6691	0.7431	0.9004	1.536	88	0.9994	0.0349	28.64
.751	43	0.6820	0.7314	0.9325	1.553	89	0.9998	0.0175	57.29
.768	44	0.6947	0.7193	0.9657	1.571	90	1.0000	0.0000	∞
.785	45	0.7071	0.7071	1.0000					

