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Global bifurcation and positive solution for a class of fully nonlinear problems[☆]

Guowei Dai[a,*], Haiyan Wang[b], Bianxia Yang[c]

[a] School of Mathematical Sciences, Dalian University of Technology, Dalian, 116024, PR China
[b] School of Mathematical and Natural Sciences, Arizona State University, United States
[c] School of Mathematics and Statistics, Lanzhou University, Lanzhou, 730000, PR China

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ABSTRACT

In this paper, we study global bifurcation phenomena for the following Kirchhoff type problem

$$\begin{cases} -M\left(\int_\Omega |\nabla u(x)|^2\,dx\right)\Delta u = \lambda f(x,u) & \text{in } \Omega,\\ u=0 & \text{on } \partial\Omega, \end{cases}$$

where M is a continuous function. Under some natural hypotheses, we show that $(\lambda_1(a)M(0), 0)$ is a bifurcation point and there is a global continuum $\mathcal{C}$ emanating from $(\lambda_1(a)M(0), 0)$, where $\lambda_1(a)$ denotes the first eigenvalue of the above problem with $f(x,s) = a(x)s$. As an application of the above result, we study the existence of positive solution for this problem with asymptotically linear nonlinearity.

1. Introduction

Consider the following Kirchhoff type problem

$$\begin{cases} -M\left(\int_\Omega |\nabla u(x)|^2\,dx\right)\Delta u = \lambda a(x)u(x) + g(x,u,\lambda) & \text{in } \Omega,\\ u=0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$, M is a continuous function on $\mathbb{R}^+$, $a \in L^\infty(\Omega)$ with $a \not\equiv 0$, $\lambda > 0$ is a parameter, $g : \Omega \times \mathbb{R}^2 \to \mathbb{R}$ satisfies the carathéodory condition in the first two variables and

$$\lim_{s\to 0} \frac{g(x,s,\lambda)}{s} = 0 \tag{1.2}$$

uniformly for a.e. $x \in \Omega$ and λ on bounded sets. Moreover, we also assume that g satisfies the growth restriction

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* Corresponding author.
E-mail address: daiguowei@dlut.edu.cn (G. Dai).

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(G) There exist $c > 0$ and $p \in (1, 2^*)$ such that

$$|g(x, s, \lambda)| \le c\left(1 + |s|^{p-1}\right)$$

for a.e. $x \in \Omega$ and λ on bounded sets, where

$$2^* = \begin{cases} \dfrac{2N}{N-2}, & \text{if } N > 2, \\ +\infty, & \text{if } N \le 2. \end{cases}$$

The problem (1.1) is nonlocal as the appearance of the term $\int_\Omega |\nabla u(x)|^2\, dx$ which implies that it is not a pointwise identity. This causes some mathematical difficulties which make the study of problem (1.1) particularly interesting. Moreover, problem (1.1) is related to the stationary problem of a model introduced by Kirchhoff in 1883 to describe the transversal oscillations of a stretched string [1]. After the famous paper by Lions [2], this type of problems has been the subject of numerous studies, and some important and interesting results have been obtained, for example, see [3–6]. Recently, there are many mathematicians studying this kind of problems by variational method, see [7–13] and the references therein. We refer to [14–20] for Kirchhoff models with critical exponents. For evolution problems, we refer to [21–23] and the references therein.

To the best of our knowledge, there are few papers that studied Kirchhoff type problems using the bifurcation theory, see for example [24,25]. The first aim of this paper is to study global bifurcation phenomena for problem (1.1). Let $\lambda_1(a)$ denote the first eigenvalue of the following problem

$$\begin{cases} -\Delta u = \lambda a(x) u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases} \tag{1.3}$$

It is well known that $\lambda_1(a)$ is simple, isolated and is the unique principle eigenvalue of problem (1.3). Now, we make the following assumptions on M.

(M_0) M is a continuous function on $\mathbb{R}^+$ such that for some $m_0 > 0$, we have

$$M(t) \ge m_0, \quad \text{for all } t \in \mathbb{R}^+;$$

(M_1) there exists $m_1 > 0$, such that $\lim_{t\to+\infty} M(t) = m_1$.

The hypothesis (M_0) shows that our problem is non-degenerate. In [14,16] the so-called "degenerate" case is covered (see also [22,23,20]), that is the main Kirchhoff non-negative function M could be zero at 0.

Our first main result is the following theorem.

Theorem 1.1. *Assume that (1.2), (G) and (M_0) hold. Then $(\lambda_1(a)M(0), 0)$ is a bifurcation point of problem (1.1) and the associated bifurcation continuum $\mathcal{C}$ in $\mathbb{R} \times H_0^1(\Omega)$, whose closure contains $(\lambda_1(a)M(0), 0)$, is either unbounded or contains a pair $(\mu M(0), 0)$, where μ is another eigenvalue of problem (1.3).*

On the basis of Theorem 1.1, the second aim of this paper is to determine the interval of λ, for which there exists a positive solution for the following Kirchhoff type problem

$$\begin{cases} -M\left(\displaystyle\int_\Omega |\nabla u(x)|^2\, dx\right)\Delta u = \lambda f(x, u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.4}$$

where $f \in C\left(\overline{\Omega} \times \mathbb{R}\right)$ satisfies that

(f_1) $f : \Omega \times \mathbb{R}^+ \to \mathbb{R}^+$ such that $f(x, s)s > 0$ for $x \in \Omega$ and any $s > 0$;

(f_2) $\lim_{s\to0^+} \frac{f(x,s)}{s} = a(x)$, $\lim_{s\to+\infty} \frac{f(x,s)}{s} = c(x) \not\equiv 0$ uniformly in $x \in \Omega$, where $a(x), c(x)$ such that they are strict positive on some subset of positive measure in Ω and $\lambda_1(c)m_1 \ne \lambda_1(a)M(0)$.

The following theorem is our second main result.

Theorem 1.2. *Suppose that (M_0)–(M_1) and (f_1)–(f_2) hold, then for*

$$\lambda \in \left(\min\{\lambda_1(c)m_1, \lambda_1(a)M(0)\},\ \max\{\lambda_1(c)m_1, \lambda_1(a)M(0)\}\right),$$

problem (1.4) possesses at least one positive solution.

Remark 1.3. Note that the corresponding existence result of [7] is a corollary of Theorem 1.2. In fact, by the monotonicity of eigenvalue with respect to weight, we get $1 \in (\lambda_1(a)M(0), \lambda_1(c)m_1)$ under the assumptions of Theorem 1 in [7]. So problem (1.4) with $\lambda = 1$ possesses at least one positive solution. Clearly, our assumptions are weaker than corresponding ones of [7]. Therefore, we improve and extend the corresponding result of [7].

The rest of this paper is organized as follows. Sections 2 and 3 present the proofs of Theorems 1.1 and 1.2, respectively.

2. Global bifurcation

Firstly, we study the global bifurcation phenomena for the following fully nonlinear abstract operator equation

$$u = F(\lambda, u), \quad u \in X \tag{2.1}$$

where λ varies in $\mathbb{R}$, X is a real Banach space with norm $\|\cdot\|$, $F : \mathbb{R} \times X \to X$ is completely continuous. Moreover, we assume that there exists a linear, completely continuous operator L such that

$$F(\lambda, u) - \lambda Lu = o(\|u\|)$$

for $u \in X$ near 0 uniformly on bounded λ intervals.

Let $r(L)$ denote the set of real characteristic values of L, and $\mathfrak{S}$ be the closure of the set of nontrivial solution pairs of Eq. (2.1). Using Theorem 1.3 of [26], we can easily get the following lemma.

Lemma 2.1. *If $\mu \in r(L) \setminus \{0\}$ has odd algebraic multiplicity, then $\mathfrak{S}$ possesses a component $\mathscr{C}_\mu$ such that $(\mu, 0) \in \mathscr{C}_\mu$ and $\mathscr{C}_\mu$ either*

(i) *meets infinity in $\mathbb{R} \times X$, or*
(ii) *meets $(\widehat{\mu}, 0)$, where $\widehat{\mu}$ is another characteristic value of L.*

Clearly, problem (1.1) can be equivalently written as

$$u = (-\Delta)^{-1}\left(\frac{1}{M\left(\int_\Omega |\nabla u|^2\,dx\right)}(\lambda a u + H(\lambda, u))\right)$$

where $H(\lambda, \cdot)$ denotes the usual Nemitsky operator associated with g. We write $X := H_0^1(\Omega)$ with the norm $\|u\| = \left(\int_\Omega |\nabla u|^2\,dx\right)^{1/2}$.

Proof of Theorem 1.1. Let

$$Lu = \frac{(-\Delta)^{-1}(au)}{M(0)}, \qquad \widetilde{H}(\lambda, u) = \frac{(-\Delta)^{-1}(H(\lambda, u))}{M\left(\|u\|^2\right)} + \frac{\lambda\left(M(0) - M\left(\|u\|^2\right)\right)}{M(0)M\left(\|u\|^2\right)}(-\Delta)^{-1}(au).$$

Clearly, $L : X \to X$ is linear completely continuous. From condition (G), (M_0) and noting $2 < 2^*$, we can see that $\widetilde{H} : \mathbb{R} \times X \to X$ is completely continuous. Moreover, it is easy to see that $\lambda_1(a)M(0)$ is a simple characteristic value of L. Then

$$(-\Delta)^{-1}\left(\frac{1}{M\left(\|u\|^2\right)}(\lambda u + H(\lambda, u))\right) = \lambda Lu + \widetilde{H}(\lambda, u).$$

Next, we show that $\widetilde{H} = o(\|u\|)$ at $u = 0$ uniformly on bounded λ intervals. It is sufficient to show that

$$\lim_{\|u\|\to 0} \frac{H(x, u)}{\|u\|} = 0 \quad \text{in } L^{p'}(\Omega).$$

Without loss of generality, we may assume that $p > 2$. Otherwise, we can consider $\tilde{p} = cp$, $c > 1$ such that $\tilde{p} \in (2, 2^*)$. From $p < 2^*$, we can see that

$$\frac{p'(p-2)}{2^*} < \frac{2^* - p'}{2^*}.$$

So we can choose a real number $r > 1$ such that

$$\frac{p'(p-2)}{2^*} \le \frac{1}{r} \le \frac{2^* - p'}{2^*}.$$

It follows that

$$p'r(p-2) \le 2^* \quad \text{and} \quad p'r' \le 2^*. \tag{2.2}$$

For any $\varepsilon > 0$, in view of (1.2) and (G), we can choose positive numbers $\delta = \delta(\varepsilon)$ and $M = M(\delta)$ such that for a.e. $x \in \Omega$, the following relations hold:

$$\left|\frac{g(x, s, \lambda)}{s}\right| \le \varepsilon \quad \text{for } 0 < |s| \le \delta,$$

$$\left|\frac{g(x, s, \lambda)}{s}\right| \le M|s|^{p-2} \quad \text{for } |s| > \delta.$$

Then we can obtain that

$$\int_{\Omega}\left|\frac{H(\lambda,u)}{u}\right|^{p'r}dx\le\varepsilon|\Omega|+M^{p'r}\int_{\Omega}|u|^{p'r(p-2)}\,dx.$$

From this inequality, (2.2) and $u\to 0$ in X, we get that

$$\left|\frac{H(\lambda,u)}{u}\right|^{p'}\to 0\quad\text{in }L^{r}(\Omega).\tag{2.3}$$

Let $v=u/\|u\|$. By the boundedness of v in X, (2.2) and the continuous embedding of $X\hookrightarrow L^{2^*}(\Omega)$, we have that

$$\int_{\Omega}|v|^{p'r'}\,dx\le c\tag{2.4}$$

for some constant $c>0$. Then from (2.3), (2.4) and Hölder's inequality, we obtain that

$$\begin{aligned}\int_{\Omega}\left|\frac{H(\lambda,u)}{\|u\|}\right|^{p'}dx&=\int_{\Omega}\left|\frac{H(\lambda,u)}{|u|}\right|^{p'}|v|^{p'}\,dx\\&\le\left(\int_{\Omega}\left|\frac{H(\lambda,u)}{u}\right|^{p'r}dx\right)^{1/r}\left(\int_{\Omega}|v|^{p'r'}\,dx\right)^{1/r'}\\&\to 0.\end{aligned}$$

Now, from Lemma 2.1, we get the existence of a global branch of the set of nontrivial solution of problem (1.1) emanating from $(\lambda_1(a)M(0),0)$. ■

3. Positive solution

In this section, based on the Theorem 1.1, we study the existence of positive solution for problem (1.4).

Lemma 3.1. *Assume that* (M_0) *and* (f_1)–(f_2) *hold. Then* $(\lambda_1(a)M(0),0)$ *is a bifurcation point of problem* (1.4) *and the associated bifurcation branch* $\mathcal{C}$ *in* $\mathbb{R}\times X$ *whose closure contains* $(\lambda_1(a)M(0),0)$*, is either unbounded or contains a pair* $(\overline{\mu}M(0),0)$ *in which* $\overline{\mu}$ *is another eigenvalue of problem* (1.3).

Proof. Let $\vartheta:\Omega\times\mathbb{R}^+\to\mathbb{R}^+$ be a continuous function such that

$$f(x,u)=a(x)u+\vartheta(x,u)$$

with

$$\lim_{s\to 0^+}\frac{\vartheta(x,s)}{s}=0\quad\text{and}\quad\lim_{s\to+\infty}\frac{\vartheta(x,s)}{s}=c(x)-a(x)\text{ uniformly in }\Omega.\tag{3.1}$$

From (3.1), we can see that $\lambda\vartheta(x,u)$ satisfies the hypotheses of (1.2) and (G). Now, Theorem 1.1 can be applied to get the desired results. ■

Next, we shall prove that the first choice of the alternatives of Lemma 3.1 is valid. Let P denote the set of functions in X which are positive in Ω. Let $K=\mathbb{R}\times P$ under the product topology.

Lemma 3.2. *We have* $\mathcal{C}\subseteq(K\cup\{(\lambda_1(a)M(0),0)\})$ *and the last alternative of* Lemma 3.1 *is impossible.*

Proof. Clearly, we have $u\ge 0$ for any nontrivial solution $(\lambda,u)\in\mathcal{C}$ because $f\ge 0$. By the strong maximum principle [27], we know that $u>0$ in the whole domain for any nontrivial solution $(\lambda,u)\in\mathcal{C}$. So we have $\mathcal{C}\subseteq(K\cup(\mathbb{R}\times\{0\}))$. Suppose on the contrary, if there exists $(\lambda_m,u_m)\to(\mu M(0),0)$ when $m\to+\infty$ with $(\lambda_m,u_m)\in\mathcal{C}$, $u_m\not\equiv 0$ and $\mu\ne\lambda_1(a)$. Let $v_m:=u_m/\|u_m\|$, then v_m satisfies

$$v_m=(-\Delta)^{-1}\left(\frac{\lambda_m a(x)v_m(x)}{M(\|u_m\|)}+\frac{\lambda_m\vartheta(x,u_m(x))}{M(\|u_m\|)\,\|u_m\|}\right).$$

By an argument similar to that of Theorem 1.1, we obtain that for some convenient subsequence, $v_m\to v_0\ne 0$ as $m\to+\infty$. It is easy to see that v_0 verifies the equation

$$\begin{cases}-\Delta v_0(x)=\mu a(x)v_0 & \text{in }\Omega,\\ v_0=0 & \text{on }\partial\Omega.\end{cases}$$

So it follows from [28] that v_0 must change its sign. This is a contradiction. ■

Proof of Theorem 1.2. We only need to show $\mathscr{C}$ links $(\lambda_1(a)M(0), 0)$ to $(\lambda_1(c)m_1, +\infty)$ in $\mathbb{R} \times X$. Let $(\lambda_n, u_n) \in \mathscr{C}$ with $u_n \not\equiv 0$ satisfies

$$\lambda_n + \|u_n\| \to +\infty.$$

We note that $\lambda_n > 0$ for all $n \in \mathbb{N}$, since $(0, 0)$ is the only solution of (1.4) for $\lambda = 0$ and $\mathscr{C} \cap (\{0\} \times X) = \emptyset$.

We claim that there exists a constant $M > 0$ such that

$$\lambda_n \subset (0, M]$$

for $n \in \mathbb{N}$ large enough. On the contrary, we suppose that $\lim_{n\to\infty} \lambda_n \to +\infty$. Since $(\lambda_n, u_n) \in \mathscr{C}$, it follows that

$$-\Delta u_n(x) = \frac{\lambda_n}{M(\|u_n\|)} \frac{f(x, u_n)}{u_n} u_n \quad \text{in } \Omega.$$

It follows from (M_0)–(M_1) that there exists $C > 0$ such that $\frac{1}{M(\|u_n\|)} \geq 1/C$. It follows from (f_2) that there exists some open subset Ω_0 of positive measure in Ω such that a, c are positive on $\overline{\Omega}_0$. Furthermore, there exist two positive constants δ_1, δ_2 such that $\delta_1 < \delta_2$,

$$\frac{f(x, s)}{s} > \frac{a(x)}{2} \quad \text{for } s \in (0, \delta_1), \ x \in \overline{\Omega}_0$$

and

$$\frac{f(x, s)}{s} > \frac{c(x)}{2} \quad \text{for } s \in (\delta_2, +\infty), \ x \in \overline{\Omega}_0.$$

Let

$$\sigma_1 = \min_{\overline{\Omega}_0 \times [\delta_1, \delta_2]} \frac{f(x, s)}{s}.$$

Then (f_1) shows that $\sigma_1 > 0$. Clearly, we have $a, c \in C\left(\overline{\Omega}\right)$. Let

$$\sigma = \min \left\{ \delta_1, \min_{\overline{\Omega}_0} \frac{a(x)}{2}, \min_{\overline{\Omega}_0} \frac{c(x)}{2} \right\}.$$

Obviously, one has $\sigma > 0$. Then we can see that $\frac{f(x,u_n)}{u_n} \geq \sigma$ all $x \in \Omega_0, \ n \in \mathbb{N}$. Set

$$\widetilde{u}_n = \begin{cases} u_n & \text{if } x \in \Omega_0, \\ 0 & \text{if } x \in \Omega \setminus \Omega_0. \end{cases}$$

Then we have that

$$\begin{cases} -\Delta \widetilde{u}_n(x) \geq \dfrac{\sigma}{C} \lambda_n \widetilde{u}_n & \text{in } \Omega_0, \\ \widetilde{u}_n = 0 & \text{on } \partial\Omega_0. \end{cases} \tag{3.2}$$

Multiplying the first equation of problem (3.2) by a positive eigenfunction φ_1 associated to $\lambda_1(1)$, we get that

$$\lambda_1 \geq \frac{\sigma}{C} \lambda_n,$$

an absurdum.

Therefore, we get that $\|u_n\| \to \infty$. Let $\xi : \Omega \times \mathbb{R}^+ \to \mathbb{R}^+$ be a continuous function such that

$$f(x, u) = c(x)u + \xi(x, u)$$

with

$$\lim_{s\to+\infty} \frac{\xi(x, s)}{s} = 0 \quad \text{and} \quad \lim_{s\to 0^+} \frac{\xi(x, s)}{s} = a(x) - c(x) \text{ uniformly in } \Omega. \tag{3.3}$$

We divide the equation

$$\begin{cases} -\Delta u_n(x) = \dfrac{\lambda_n c(x) u_n(x)}{M(\|u_n\|)} + \dfrac{\lambda_n \xi(x, u_n(x))}{M(\|u_n\|)} & \text{in } \Omega, \\ u_n = 0 & \text{on } \partial\Omega \end{cases}$$

by $\|u_n\|$ and set $\overline{u}_n = u_n / \|u_n\|$. Since $\overline{u}_n$ is bounded in X, after taking a subsequence if necessary, we have that $\overline{u}_n \rightharpoonup \overline{u}$ for some $\overline{u} \in X$ and $\overline{u}_n \to \overline{u}$ in $L^2(\Omega)$.

It follows from (3.3) that for any $\varepsilon > 0$, there exists a constant C such that

$$|\xi (x, u_n)| \leq C + \varepsilon |u_n| . \tag{3.4}$$

By (3.4), we can easily show that

$$\lim_{n\to+\infty} \frac{\xi (x, u_n(x))}{\|u_n\|} = 0 \quad \text{in } L^2(\Omega).$$

By the compactness of $(-\Delta)^{-1} : L^2(\Omega) \to X$, we obtain

$$\begin{cases} -\Delta\overline{u}(x) = \dfrac{\overline{\lambda}}{m_1} c(x)\overline{u}(x) & \text{in } \Omega, \\ \overline{u} = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\overline{\lambda} = \lim_{n\to+\infty} \lambda_n$, again choosing a subsequence and relabeling it if necessary.

It is clear that $\overline{u} \in \overline{\mathcal{C}} \subseteq \mathcal{C}$ since $\mathcal{C}$ is closed in $\mathbb{R} \times X$. So $\overline{\lambda} = \lambda_1(c)m_1$. Therefore, $\mathcal{C}$ links $(\lambda_1(a)M(0), 0)$ to $(\lambda_1(c)m_1, \infty)$ in $\mathbb{R} \times X$. ■

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