Proceedings of the 33rd Chinese Control Conference
July 28-30, 2014, Nanjing, China

Bifurcation and Control of a Delayed Diffusive Logistic Model in Online Social Networks

ZHU Linhe[1], ZHAO Hongyong[1], WANG Haiyan[2]

1. Department of Mathematics, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
E-mail: zlhnuaa@126.com zhaohy@nuaa.edu.cn

2. School of Mathematical and Natural Sciences, Arizona State University, Phoenix AZ 85069-7100, USA
E-mail: haiyan.wang@asu.edu

Abstract: Online social networks have become a popular source for disseminating information and facilitating the building of social relations among a huge number of people. Recently, several partial differential equations were proposed to model the spatio-temporal dynamics of information diffusion in online social networks. As a result, mathematical results of reaction-diffusion equations can be used to help understand the mechanism of information diffusion and, in particular, increase the efficiency of distributing positive information while reducing unwanted information. In this paper, we develop a Partial Differential Equation (PDE) with a delayed feedback controller to effectively control the spread of harmful information. Applying the theory of partial function differential equation, we present verifiable control conditions for stability and Hopf bifurcation of the feedback control system. Examples are given to demonstrate that the delayed feedback controller can reduce the density of influenced users effectively and delay the onset of Hopf bifurcation as well.

Key Words: Online social networks, stability, Hopf bifurcation, control

1 Introduction

Online social networks have become a popular source for disseminating information and facilitating the building of social relation among a huge number of people. Understanding of information diffusion over online social networks has a significant impact on real life applications such as product marketing, political online campaign, etc. A wealth of research has studied some significant characteristics of information diffusion over various online social networks using empirical approaches [1, 2, 3, 4] . Recently, mathematical modeling has played an increasingly important role in understanding information diffusion in online social networks. Existing dynamical models of information diffusion in online social networks mainly studied information diffusion in temporal dimension [5, 6]; only a few of works have attempted to understand and model information diffusion in both temporal and spatial dimensions [7, 8].

In a recent paper [7] , it was the first time that a partial differential equation model was introduced to study the spatio-temporal diffusion problem in online social networks. In [7] , the authors used friendship hops as distance and abstractly translated the information diffusion process in online social networks into two separate processes: growth process and social process. The model in [7] was described by the following partial differential equation:

$$\begin{cases} \dfrac{\partial I}{\partial t} = d\dfrac{\partial^2 I}{\partial x^2} + rI(1-\dfrac{I}{K}) \\ I(x,1) = \phi(x), l < x < L \\ \dfrac{\partial I}{\partial x}(l,t) = \dfrac{\partial I}{\partial x}(L,t) = 0, t > 1, \end{cases} \tag{1}$$

where I, K, d, r, l and L represent the density of influenced users with a distance of x at time t, the carrying capacity, the social capability, the intrinsic growth rate of influenced users, the lower and the upper bounds of distance between the source and other social network users, respectively. In [8], experimental dates showed that the intrinsic growth rate r of system (1) was often dependent on time t. More specifically, r was a decreasing function of t. And the decay could be modeled by the following ordinary differential equation:

$$\begin{aligned} &\frac{dr}{dt} = -\alpha r(t) + \beta \\ &r(1) = \gamma, \end{aligned} \tag{2}$$

where α is the decay rate, γ is the initial rate of influence. β represents the residual rate after a news story becomes stable, which can be very small.

As is well known, in order to avoid the spread of harmful information in online social networks, the government usually takes measures to control the diffusion of the harmful information. At last, the density of influenced users can be controlled in a lower state. It was pointed out in [9, 10] that in some situations where the equilibrium point of system (1) is not the desirable one and a smaller value is required. We can alter system (1) structurally by introducing a feedback controller.

*This work is supported by National Natural Science Foundation (NNSF) of China under Grant 61174155, 11032009

On the other hand, the government sometimes can not take notice of the perniciousness of the information promptly. Without loss of generality, we assume that the government feedback mechanism takes τ unites of time to respond.

Motivated by the above discussions, in this paper we will investigate the partial differential equation model with a delayed feedback controller for online social networks. The system is given in the following form:

$$\begin{cases} \dfrac{\partial I}{\partial t} = d\dfrac{\partial^2 I}{\partial x^2} + rI(1-\dfrac{I}{K} - cu(t-\tau)), t>0, 0<x<\pi \\ \dfrac{dr}{dt} = -\alpha r + \beta \\ \dfrac{du}{dt} = -au + bI \\ r(t)=\gamma, u(t)=\mu, I(t,x)=\phi(t,x), (t,x)\in[-\tau,0]\times[0,\pi], \end{cases} \tag{3}$$

with the Neumann boundary condition

$$\frac{\partial I}{\partial x}(0,t) = \frac{\partial I}{\partial x}(\pi,t) = 0, t\geq 0,$$

where the meanings of I, K, d, r, α and β are the same as [7]. u is a government feedback controller. τ denotes that the feedback mechanism takes τ unites of time to respond, and a, b and c are positive parameters. In this paper we assume that γ, μ and $\phi(t,x)\in C = C([-\tau,0],X)$ are initial conditions, and denote $X = \{I\in W^{2,2}(0,\pi) : I_x(0) = I_x(\pi) = 0\}$ with the inner product $<\cdot,\cdot>$. The Neumann boundary condition $\frac{\partial I}{\partial x}(0,t) = \frac{\partial I}{\partial x}(\pi,t) = 0$ means no flux of information across the boundaries, which is plausible for online social networks since information spreads within a few of friendship hops of the networks.

The structure of this paper is arranged as follows. In Section 2, we study the local stability and the existence of Hopf bifurcation at the equilibrium point. In Section 3, some numerical simulations are given to support the analysis and theoretical predictions of Section 2.

2 Local stability and Hopf bifurcation

In this section we discuss the local stability and Hopf bifurcation of system (3) by analyzing the corresponding characteristic equation.

Through a simple calculation, we can obtain that system (3) has a unique positive equilibrium point $E^* = (I^*, r^*, u^*)^T$, where

$$I^* = \frac{aK}{a+bcK},\ r^* = \frac{\beta}{\alpha},\ u^* = \frac{bK}{a+bcK}.$$

Let $\overline{I} = I - I^*$, $\overline{r} = r - r^*$, $\overline{u} = u - u^*$ and drop bars for simplicity of notations. Then system (3) can be rewritten as the following form:

$$\begin{cases} \dfrac{\partial I}{\partial t} = d\dfrac{\partial^2 I}{\partial x^2} + (r^* - \dfrac{2r^*I^*}{K} - cu^*r^*)I + (I^* - \dfrac{I^{*2}}{K} - cu^*I^*)r \\ -cr^*I^*u(t-\tau) + f, t>0, 0<x<\pi \\ \dfrac{dr}{dt} = -\alpha r \\ \dfrac{du}{dt} = -au + bI, \end{cases} \tag{4}$$

where

$$f = (1-\frac{2I^*}{K} - cu^*)rI - cI^*ru(t-\tau) - cr^*Iu(t-\tau)$$
$$-cru(t-\tau)I - \frac{r^*}{K}I^2 - \frac{1}{K}rI^2.$$

Thus, the positive equilibrium point E^* of system (3) is transformed into the zero equilibrium point $E^0 = (0,0,0)^T$ of system (4). Let

$$U(t) = (u_1(t), u_2(t), u_3(t))^T = (I(t,\cdot), r(t), u(t))^T,$$

then (4) can be rewritten as an abstract differential equation in the phase space $C = C([-\tau,0],X)$ of the form:

$$\dot{U} = D\Delta U(t) + L(U_t) + f(U_t), \tag{5}$$

where

$$D = diag\{d,0,0\},$$

$$\Delta = diag\{\partial^2/\partial x^2, 0, 0\},$$

$$L : C \to X$$

and

$f : C \to X$ are given, respectively, by

$$L(\varphi) = ((r^* - \frac{2r^*I^*}{K} - cu^*r^*)\varphi_1(0) + (I^* - \frac{I^{*2}}{K} - cu^*I^*)\varphi_2(0)$$
$$-cr^*I^*\varphi_3(-\tau), -\alpha\varphi_2(0), b\varphi_1(0) - a\varphi_3(0))^T$$

and

$$f(\varphi) = ((1-\frac{2I^*}{K} - cu^*)\varphi_1(0)\varphi_2(0) - \frac{r^*}{K}\varphi_1^2(0)$$
$$-\frac{1}{K}\varphi_2(0)\varphi_1^2(0) - cr^*\varphi_1(0)\varphi_3(-\tau)$$
$$-cI^*\varphi_2(0)\varphi_3(-\tau) - c\varphi_1(0)\varphi_2(0)\varphi_3(-\tau), 0, 0)^T.$$

For $\varphi(\theta) = U_t(\theta)$, $\varphi = (\varphi_1, \varphi_2, \varphi_3)^T \in C$, the linearzed system of (5) at E^0 is

$$\dot{U} = D\Delta U(t) + L(U_t) \tag{6}$$

and its characteristic equation is

$$\lambda y - D\Delta y - L(e^{\lambda \cdot} y) = 0 \quad (7)$$

where $y \in dom(\Delta)$, and $y \neq 0$, $dom(\Delta) \subset X$.

From the properties of the Laplacian operator defined on the bounded domain, the operator Δ on X has the eigenvalues $-k^2, k \in N_0 = \{0,1,2\cdots\}$ with the corresponding eigenfunctions on X are

$$\beta_k^1 = (\gamma_k, 0, 0)^T, \beta_k^2 = (0, \gamma_k, 0)^T, \beta_k^3 = (0, 0, \gamma_k)^T, \quad (8)$$

where $\gamma_k = \cos(kx)$ and $(\beta_k^1, \beta_k^2, \beta_k^3)_0^\infty$ constructs a basis of the phase space X. Therefore any element y in X can be expanded as Fourier series in the following form

$$y = \sum_{k=0}^{\infty} Y_k^T (\beta_k^1, \beta_k^2, \beta_k^3)^T, Y_k = (< y, \beta_k^1 >, < y, \beta_k^2 >, < y, \beta_k^2 >)^T. \quad (9)$$

By a simple calculation, we obtain

$$L(\varphi^T (\beta_k^1, \beta_k^2, \beta_k^3)^T) = L(\varphi^T)(\beta_k^1, \beta_k^2, \beta_k^3)^T, k \in N_0. \quad (10)$$

From (9) and (10), we know that the characteristic roots of (7) are given by the following sequence of characteristic equations

$$\sum_{k=0}^{\infty} Y_k^T \left[\lambda I_3 + Dk^2 - \begin{pmatrix} r^* - \frac{2r^* I^* + u^* r^* kc}{k} & 0 & b \\ I^* - \frac{I^{*2}}{k} - u^* I^* c & -\alpha & 0 \\ -r^* I^* c e^{-\lambda\tau} & 0 & -a \end{pmatrix} \right] \begin{pmatrix} \beta_k^1 \\ \beta_k^2 \\ \beta_k^3 \end{pmatrix} = 0, \quad (11)$$

Hence, we conclude that the characteristic equation (11) about E^0 takes the form

$$\begin{aligned} &(\lambda + \alpha)[\lambda^2 + (dk^2 - r^* + \frac{2r^* I^*}{K} + cr^* u^* + a)\lambda + \\ &(adk^2 - ar^* + \frac{2ar^* I^*}{K} + acr^* u^*) + bcr^* I^* e^{-\lambda\tau}] = 0. \end{aligned} \quad (12)$$

From $\alpha > 0$, then Eq. (12) can be written as

$$\begin{aligned} &\lambda^2 + (dk^2 - r^* + \frac{2r^* I^*}{K} + cr^* u^* + a)\lambda + \\ &(adk^2 - ar^* + \frac{2ar^* I^*}{K} + acr^* u^*) + bcr^* I^* e^{-\lambda\tau} = 0. \end{aligned} \quad (13)$$

Eq. (13) with $\tau = 0$ is equivalent to the following quadratic equation with respect to λ

$$\lambda^2 + (dk^2 + a + \frac{r^* I^*}{K})\lambda + (adk^2 + \frac{ar^* I^*}{K} + bcr^* I^*) = 0. \quad (14)$$

In the section, we make the following assumes

(H1) $\frac{a^2}{K^2} - b^2 c^2 < 0$;

(H2) $ad + (\frac{a}{K} - bc) r^* I^* > 0$.

Theorem 1 For arbitrary $a, b, c, d, K, \alpha, \beta > 0$, the zero equilibrium point E^0 of system (4) with $\tau = 0$ is locally asymptotically stable.

Proof. Clearly, from (14) it follows that

$$dk^2 + a + \frac{r^* I^*}{K} \geq a + \frac{r^* I^*}{K} > 0$$

and

$$adk^2 + \frac{ar^* I^*}{K} + bcr^* I^* \geq \frac{ar^* I^*}{K} + bcr^* I^* > 0.$$

By the Routh-Hurwitz criteria, all the roots of Eq. (14) have negative real parts. Therefore, we have the above result.

Theorem 2 Assume (H1) and (H2) hold. Then for system (4), the following statements are true.

(i) When $\tau \in [0, \tau_0^0)$, then the zero equilibrium point E^0 is locally asymptotically stable.

(ii) The Hopf bifurcation occurs when $\tau = \tau_0^j$. That is, system (4) has a branch of periodic solutions bifurcating from the zero equilibrium point E^0 near $\tau = \tau_0^j$, where

$$\tau_0^j = \frac{1}{\omega}(\arccos \frac{\omega^2 - \frac{ar^* I^*}{K}}{bcr^* I^*} + 2j\pi), j = 0, 1, 2\cdots,$$

$$\omega = \sqrt{\frac{-a^2 - \frac{r^{*2} I^{*2}}{K^2} + \sqrt{(a^2 + \frac{r^{*2} I^{*2}}{K^2})^2 + 4b^2 c^2 r^{*2} I^{*2}}}{2}}.$$

Proof. Now we discuss the effect of the delay τ on the stability of E^0. Assume that $i\omega$ is a root of (13). Then ω should satisfy the following equation

$$-\omega^2 + i\omega(dk^2 + a + \frac{r^* I^*}{K}) + (adk^2 + \frac{ar^* I^*}{K}) + bcr^* I^* e^{-i\omega\tau} = 0. \quad (15)$$

Separating the real and imaginary parts, we have

$$\begin{cases} bcr^*I^*\sin\omega\tau = \omega(dk^2 + a + \frac{r^*I^*}{K}) \\ bcr^*I^*\cos\omega\tau = \omega^2 - dk^2 - \frac{ar^*I^*}{K}, \end{cases} \quad (16)$$

taking square on both sides of the equations of (16) and summing them up, we obtain

$$\omega^4 + (d^2k^4 + \frac{2r^*I^*}{K}dk^2 + a^2 + \frac{r^{*2}I^{*2}}{K^2})\omega^2 + a^2d^2k^4 + \frac{2a^2r^*I^*}{K}dk^2 + \frac{a^2r^{*2}I^{*2}}{K^2} - b^2c^2r^{*2}I^{*2} = 0. \quad (17)$$

When $k = 0$, Eq. (17) becomes

$$\omega^4 + (a^2 + \frac{r^{*2}I^{*2}}{K^2})\omega^2 + \frac{a^2r^{*2}I^{*2}}{K^2} - b^2c^2r^{*2}I^{*2} = 0 \quad (18)$$

Setting $z = \omega^2$, Eq. (17) is transformed into the following form

$$z^2 + (d^2k^4 + \frac{2r^*I^*}{K}dk^2 + a^2 + \frac{r^{*2}I^{*2}}{K^2})z + a^2d^2k^4 + \frac{2a^2r^*I^*}{K}dk^2 + \frac{a^2r^{*2}I^{*2}}{K^2} - b^2c^2r^{*2}I^{*2} = 0. \quad (19)$$

and we can rewrite Eq. (18) as

$$z^2 + (a^2 + \frac{r^{*2}I^{*2}}{K^2})z + \frac{a^2r^{*2}I^{*2}}{K^2} - b^2c^2r^{*2}I^{*2} = 0. \quad (20)$$

According to (H1), Eq. (20) has a unique positive solution

$$z = \frac{-a^2 - \frac{r^{*2}I^{*2}}{K^2} + \sqrt{(a^2 + \frac{r^{*2}I^{*2}}{K^2})^2 + 4b^2c^2r^{*2}I^{*2}}}{2}.$$

That is, $\omega = \sqrt{\frac{-a^2 - \frac{r^{*2}I^{*2}}{K^2} + \sqrt{(a^2 + \frac{r^{*2}I^{*2}}{K^2})^2 + 4b^2c^2r^{*2}I^{*2}}}{2}}$

is the unique positive solution of Eq. (19).

Next, we can validate that ω is a simple solution.

When $k = 0$, Eq. (13) becomes the following form

$$\lambda^2 + (\frac{r^*I^*}{K} + a)\lambda + \frac{ar^*I^*}{K} + bcr^*I^*e^{-\lambda\tau} = 0. \quad (21)$$

Differentiating the two sides of (21) with respect to λ yields

$$2\lambda + a + \frac{r^*I^*}{K} - bc\tau r^*I^*e^{-\lambda\tau} = 0. \quad (22)$$

From (21), simple computation shows that

$$e^{-\lambda\tau} = -\frac{\lambda^2 + (\frac{r^*I^*}{K} + a)\lambda + \frac{ar^*I^*}{K}}{bcr^*I^*}. \quad (23)$$

Submitting (23) into (22) yields

$$\tau\lambda^2 + (\tau a + \frac{\tau r^*I^*}{K} + 1)\lambda + (\frac{\tau ar^*I^*}{K} + \frac{r^*I^*}{K} + a) = 0. \quad (24)$$

If ω is not a simple solution, then for $\lambda = i\omega$, the following equation holds

$$-\tau\omega^2 + i\omega(\tau a + \frac{\tau r^*I^*}{K} + 1) + (\frac{\tau ar^*I^*}{K} + \frac{r^*I^*}{K} + a) = 0,$$

which implies that

$$\begin{cases} -\tau\omega^2 + \frac{\tau ar^*I^*}{K} + \frac{r^*I^*}{K} + a = 0 \\ \omega(\tau a + \frac{\tau r^*I^*}{K} + 1) = 0. \end{cases}$$

Obviously, the above equation set has no solutions. Therefore ω is a simple solution.

According to (16), we have

$$\tau_0^j = \frac{1}{\omega}(\arccos\frac{\omega^2 - \frac{ar^*I^*}{K}}{bcr^*I^*} + 2j\pi), j = 0,1,2\cdots.$$

When $k = 1$, under the assumption (H2), we have

$$a^2d^2 + \frac{2a^2r^*I^*}{K}d + \frac{a^2r^{*2}I^{*2}}{K^2} - b^2c^2r^{*2}I^{*2} = [ad + (\frac{a}{K} + bc)r^*I^*][ad + (\frac{a}{K} - bc)r^*I^*] > 0.$$

Hence, when $k \geq 1$, it is easy to show that

$$a^2d^2k^4 + \frac{2a^2r^*I^*}{K}dk^2 + \frac{a^2r^{*2}I^{*2}}{K^2} - b^2c^2r^{*2}I^{*2} \geq a^2d^2 + \frac{2a^2r^*I^*}{K}d + \frac{a^2r^{*2}I^{*2}}{K^2} - b^2c^2r^{*2}I^{*2} > 0$$

and

$$d^2k^4 + \frac{2r^*I^*}{K}dk^2 + a^2 + \frac{r^{*2}I^{*2}}{K^2} > 0.$$

By the Routh-Hurwitz criteria, all the roots of Eq. (18) have negative real parts. That is, Eq. (17) has no positive roots for $\forall k \geq 1$.

Differentiating the two sides of (21) with respect to τ yields

$$\mathrm{Re}[\frac{d\lambda}{d\tau}]^{-1}\Big|_{\lambda=i\omega,\tau=\tau_0^0}=\frac{(2\lambda+a+\frac{r^*I^*}{K})e^{\lambda\tau}}{bcr^*I^*\lambda}-\frac{\tau}{\lambda}\Big|_{\lambda=i\omega,\tau=\tau_0^0}$$

$$=\frac{a\sin\omega\tau+\frac{r^*I^*}{K}\sin\omega\tau+2\omega\cos\omega\tau}{bc\omega r^*I^*}\Big|_{\tau=\tau_0^0}$$

$$=-\frac{i(2i\omega+a+\frac{r^*I^*}{K})+(\cos\omega\tau_0^0+i\sin\omega\tau_0^0)}{bc\omega r^*I^*}-\frac{\tau}{i\omega}$$

$$=\frac{2\omega^2+a^2+\frac{r^{*2}I^{*2}}{K^2}}{b^2c^2r^{*2}I^{*2}}>0.$$

Obviously, $\lambda=0$ is not the solution of Eq. (13). According to the theory of Hopf bifurcation, Theorem 2 holds.

3 Numerical Simulation

In this section, we give some example to illustrate our theoretical results, and give numerical simulations with the help of Matlab.

Example 1 Keeping the parameters as $d=0.01, K=25$, $\alpha=1.5,\beta=0.375$, which are the same with [7], and taking $a=2, b=0.1$ and $c=0.86$. Obviously, the equilibrium point of system (1) is $I=25$, and the positive equilibrium point E^* of system (3) is $(12.0482, 0.25, 0.6024)^T$. A calculation shows that the parameters satisfy the conditions of Theorem 2. According to Theorem 2, we can get the critical value is $\tau_0^0=57.8293$. As Fig.1 shown, system (3) is asymptotically stable at the positive equilibrium point E^* for $\tau=8<\tau_0^0$. This means that when the harmful information diffuses in online social networks, the delayed feedback controller can make the density of influenced users in a lower state. And at last, the density of influenced users converges to the constant steady state. That is to say, there are no users to be influenced. And as shown in Fig. 2, when $\tau=57.85>\tau_0^0$, the spatially homogeneous periodic solutions emerge from the equilibrium point E^*. This means that when the government can not control the spread of harmful information promptly, the density of influenced users will vary periodically.

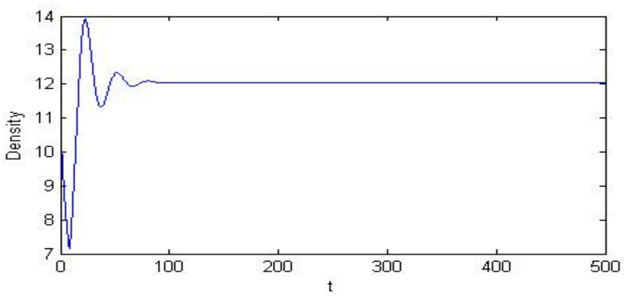

Fig.1 The density of influenced users converges to the constant steady state.

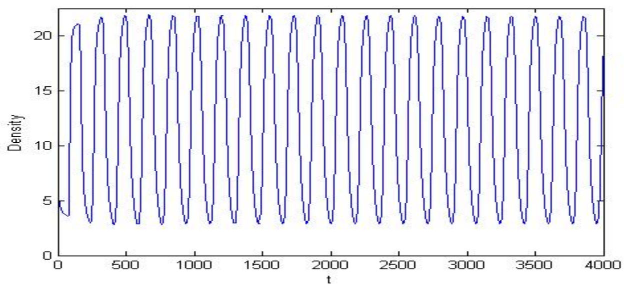

Fig. 2 The density of influenced users tends to the spatially homogenous periodic oscillation.

Example 2 Keeping the parameters as $\alpha=1.5,\beta=0.375$, $K=25$, which are the same with [7]. And we might as well take $a=3, b=0.2, d=0.1$ and c varies. A calculation shows that the parameters satisfy the conditions of Theorem 2. According to Theorem 2, we can obtain the corresponding situations of the positive equilibrium points and the corresponding critical value τ_0^0, as shown in Table 1.

From Table 1, we can see that with the increase of feedback coefficient c, the density of influenced users as well as the critical value of τ decrease together. This means that by adjusting the value of c, the government can control the density of influenced users based on actual demand. Also, when the reaction time of the government feedback mechanism is more than the critical value τ_0^0, the density of influenced users will vary periodically. To some degree, it is possible to cause social unrest.

Table 1: The positive equilibrium point E^* and the critical value τ_0^0 of system (3) vary with c increasing

c	The positive equilibrium points	τ_0^0
1.15	8.5714, 0.2500, 0.5714	14.8184
1.25	8.1081, 0.2500, 0.5495	13.6721
1.35	7.6923, 0.2500, 0.5128	12.7948

A calculation shows that the parameters satisfy the conditions of Theorem 2. Thus, the corresponding equilibrium points are locally asymptotically stable for $\tau=8<\tau_0^0$, and the density of influenced users is decreasing. See Fig. 3. This means that if the government changes control measures, the density of influenced users can be controlled in a lower state. And at last, the density of influenced users converges to the constant steady state. That is to say, there are no new users to be influenced. It is significant to maintain social stability. On the other hand, when $\tau=16>\tau_0^0$, the spatially homogeneous periodic solutions emerge from the corresponding equilibrium points, and with the decrease of the feedback coefficient c, it is not hard to see that the amplitude of the density of influenced users is decreasing, and the density of influenced users is decreasing with the increase of the feedback coefficient c, as shown in Fig. 4. Although this implies that when $\tau = 16 > \tau_0^0$, system (3) is unstable and the spatially homogeneous periodic solution emerges from the corresponding equilibrium points, but we can reduce the amplitude of the density of influenced users to lessen the dangers from the harmful information by taking the government feedback mechanism. From the different amplitudes, we conclude that it is helpful to ease social unrest when the government reduces the degree of the intervention properly. Obviously, the government feedback mechanism is significant for controlling the diffusion of information in online social networks.

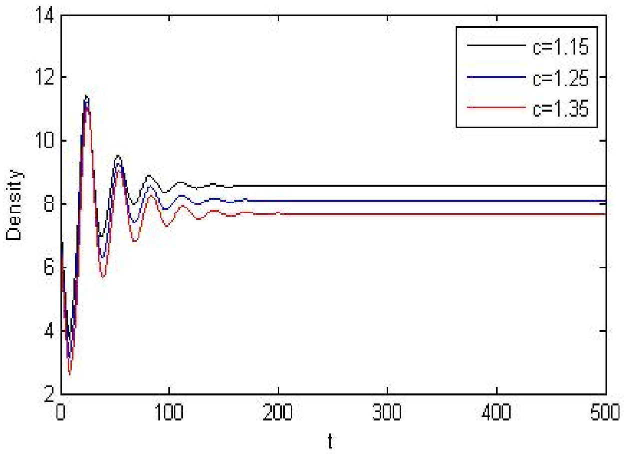

Fig. 3 The density of influenced users varies with the increase of feedback coefficient c when $\tau = 8 < \tau_0^0$.

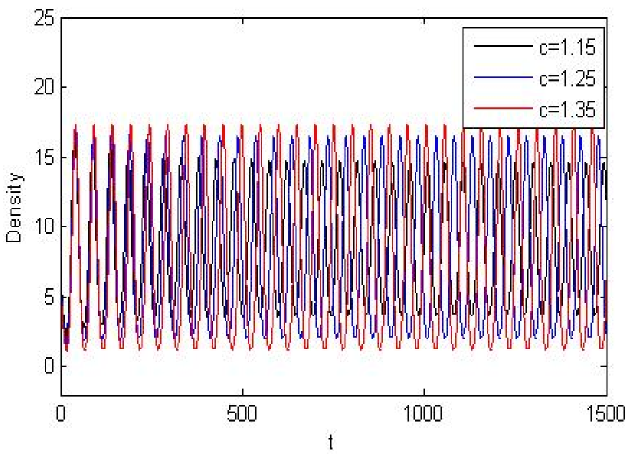

Fig. 4 The density of influenced users varies with the increase of feedback coefficient c when $\tau = 16 > \tau_0^0$.

4 Conclusions

This paper introduces a PDE with the delayed feedback controller to effectively control the spread of harmful information. Applying the theory of partial function differential equation, we present verifiable control conditions for stability and Hopf bifurcation of the feedback control system. Examples are given to demonstrate that the government feedback mechanism can effectively reduce the density of influenced users, delay the onset of Hopf bifurcation and vary the amplitude of the density of influenced users by adjusting the value of c as well. This result suggests that if the government takes effective measures to control the diffusion of the harmful information in online social networks promptly, then the density of influenced users can be controlled in a lower state, and at last the density converges to the constant steady state. That is, there are no new users to be influenced. However, sometimes the government has not become aware of the perniciousness of the information from online social networks promptly, but we also can reduce the amplitude of the density of influenced users by taking the government feedback mechanism so as to reduce the dangers from the harmful information in online social networks. As such, the feedback mechanism plays a pivotal role in information diffusion over online social networks.

References

[1] G. Steeg, R. Ghosh, and K. Lerman, What stops social epidemic?, in *Proceedings of International AAAI Conference on Weblogs and Social Media,* 2011.

[2] F. Benevenuto, Characterizing user behavior in online social networks, in *Proceedings of ACM SIGCOMM International Measurement Conference,* 2009: 49-62.

[3] K. Lerman, and R. Ghosh, Information contagion: an empirical study of spread of news on digg and twitter social networks, in: *Proceedings of International Contagion in Weblogs and Social Media,* 2010.

[4] R. Ghosh, and K. Lerman, A framework for quantitative analysis of cascades on networks, in: *Proceedings of Web Search and Data Mining Conference*, 2011: 665-674.

[5] J. Yang, and J. Leskovec, Modeling information diffusion in implicit networks, in *Proceedings of IEEE International Conference on Data Mining,* 2010: 599-608.

[6] S. Tang, N. Blenn, C. Doerr, and P. Mieghem, Digging in the Digg Social News Website, *IEEE Trans. on Multimedia,* 13(5): 1163-1175, 2011.

[7] F. Wang, H. Wang, and K. Xu, Diffusive logistic model towards predicting information diffusion in online social networks, in *32nd International Conference on Distributed Computing Systems Workshops,* 2012: 133-139.

[8] F. Wang, H. Wang, K. Xu, J. Wu, and X. Jia, Characterizing information diffusion in Online Social Networks with linear diffusive model, in *33rd International Conference on Distributed Computing Systems Workshops,* 2013: 307-316.

[9] S. Fang, and M. Jiang, Stability and Hopf bifurcation for a regulated logistic growth model with discrete and distributed delays, *Commun Nonlinear Sci Numer Simulat*, 14(12): 4292-4303, 2009.

[10] K. Gopalsamy, and P. Wang, Feedback regulation of logistic growth, *International Journal of Mathematics and Mathematical Sciences*, 16(1): 177-192, 1993.